Elementary Statistics

Lecture 9

Geometric Prob. dist:

It is similar to binomial Prob. dist. except

1) There is no fixed number of trials n.

2) x is the number of trials when first success happens. $x \geq 1$

$1, 2, 3, 4, \ldots$

$$P(x) = p \cdot q^{x-1}$$

$p \to$ Prob. of Success

$q \to$ Prob. of failure

$p + q = 1$, $q = 1 - p$

Mean $\mu = \frac{1}{p}$

Variance $\sigma^2 = \frac{q}{p^2}$

Standard Deviation $\sigma = \sqrt{\sigma^2}$

Consider a geometric prob. dist with $p = .2$

$q = 1 - p = .8$

$\sigma^2 = \frac{q}{p^2} = \frac{.8}{.2^2} = 20$

$\mu = \frac{1}{p} = \frac{1}{.2} = 5$

$\sigma = \sqrt{\sigma^2} = \sqrt{20} \approx 4.5$

Usual Range $\mu \pm 2\sigma = 5 \pm 2(4.5)$

$= 5 \pm 9 \Rightarrow \boxed{-4 \text{ to } 14}$

P(First Success happens on 3rd trial)

$P(x=3) = \underset{p}{.2} \cdot (\underset{q}{.8})^{3-1}$ $x-1$ $= .2 \cdot (.8)^2 = \boxed{.128}$

Using TI Command

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ ↓↓ $\boxed{\text{geometpdf}}(.2, 3) = \boxed{.128}$

P(First Success happens before the 3rd attempt)

$P(x < 3) = P(x \le 2) = \text{geometcdf}(.2, 2) = \boxed{.36}$

Prob. of making Sales on any call is .4.

$p = .4$ $q = .6$

$\mu = \frac{1}{p} = \frac{1}{.4} = 2.5$

$\sigma^2 = \frac{q}{p^2} = \frac{.6}{.4^2} = 3.75$

$\sigma = \sqrt{\sigma^2} = \sqrt{3.75} \approx 1.936$

Round μ & σ to a whole #, then find $\boxed{68\% \text{ Range}}$

$\mu \approx 3$, $\sigma \approx 2$

$\mu \pm \sigma = 3 \pm 2$

$\Rightarrow \boxed{1 \text{ to } 5}$

P(first Sales happens on the 4th Call)

$P(x=4) = \text{geomet pdf}(.4, 4) = .0864 \approx \boxed{.086}$

$\boxed{\frac{54}{625}}$

P(first Sales happens after the 4th Call)

$P(x > 4) = P(x \ge 5) = 1 - P(x \le 4)$

Total Prob.

$= 1 - \text{geometcdf}(.4, 4)$

$= .1296 \approx \boxed{.130}$

we don't want 4

we want 5

SG 17

$\frac{81}{625}$

Also in SG 17

Poisson Prob. dist:

Average # of Successes μ is given for a fixed interval.

x is # of Successes in that fixed interval

$x \geq 0$ $\quad 0, 1, 2, 3, 4, \ldots$

$$P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu} \qquad e \approx 2.718$$

$\sigma^2 = \mu$, $\sigma = \sqrt{\sigma^2}$

Consider a poisson prob. dist with $\mu = 16$ over a fixed interval.

$\sigma^2 = \mu = 16$ $\qquad \sigma = \sqrt{\sigma^2} = \sqrt{16} = 4$

Usual Range $\mu \pm 2\sigma = 16 \pm 2(4) = 16 \pm 8$

$\Rightarrow \boxed{8 \text{ to } 24}$

$$P(x = 15) = \frac{16^{15}}{15!} \cdot e^{-16} = \boxed{.099} \qquad e^{-\mu} \qquad e \approx 2.718$$

μ, x, $x!$

Lambda

$\mu = \lambda$

Use TI Command

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ ↓↓ poissonpdf(16, 15) = $\boxed{.099}$

x

$P(x < 20) = P(x \leq 19) = \text{Poissoncdf}(16, 19) = \boxed{.812}$

Judi draw blood in average 60 in one shift

$\mu = 60$ Fixed interval

$\sigma^2 = \mu = 60$

$\sigma = \sqrt{\sigma^2} = \sqrt{60} = 7.746$

Round μ & σ to a whole #

$\mu = 60$, $\sigma = 8$

Usual Range $\mu \pm 2\sigma = 60 \pm 2(8)$

$= 60 \pm 16 =$ 44 to 76

P(she draws 45 bloods in one shift)

$P(x=45) =$ Poisson Pdf$(60, 45) \approx \boxed{.008}$

P(she draws at least 50 bloods in one shift)

$P(x \geq 50) = 1 - P(x \leq 49)$

we don't want | we want

49 | 50

$= 1 -$ poissoncdf$(60, 49)$

↑

Total Prob.

$= \boxed{.916}$

SG 17

Working with Continuous Random Variable

Standard Normal Prob. dist:

1) Use Z-variable, $P(Z=c)=0$

2) Data dist. is symmetric, bell-shape, with total area $=1$.

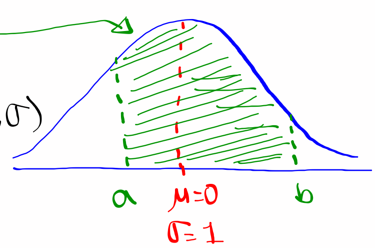
3) Mean = Mode = Median

4) $\mu = 0$, $\sigma = 1$

5) $\boxed{P(a < Z < b)}$ is the Corresponding area within the bell-shape graph.

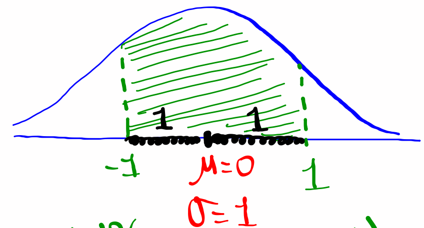
$\boxed{2nd}$ $\boxed{VARS}$

normalcdf(L, U, μ, σ)



$P(-1 < Z < 1)$

Drawing,
labeling,
Shading,
TI Command required.



$= \text{normalcdf}(-1, 1, 0, 1)$

(−)

$= \boxed{.683} \approx 68\%$

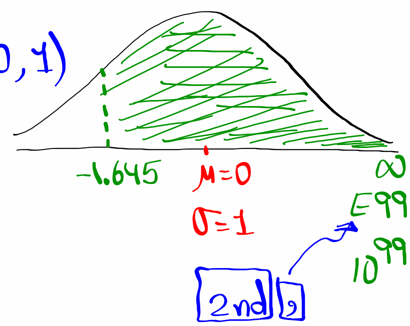
Find $P(Z > -1.645)$

$= \text{normalcdf}(-1.645, E99, 0, 1)$

(−)

$\boxed{2nd}\boxed{,}$

$= \boxed{.950}$



Find $P(Z < 1.96)$

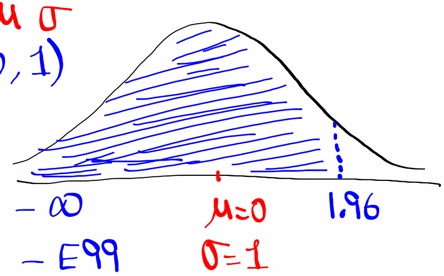
L U μ σ

$= \text{normalcdf}(-E99, 1.96, 0, 1)$

(−)

$\boxed{2nd}\boxed{,}$

$= \boxed{.975}$

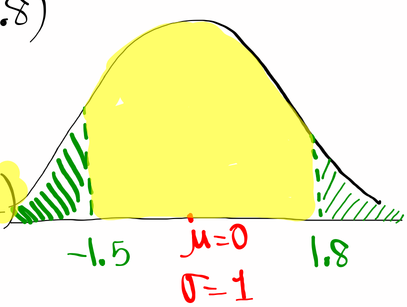


$P(Z < -1.5 \text{ OR } Z > 1.8)$

$= 1 - P(-1.5 < Z < 1.8)$

$= 1 - \text{normalcdf}(-1.5, 1.8, 0, 1)$

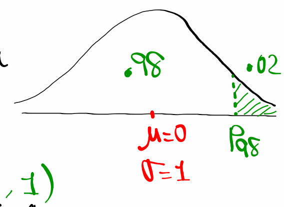
$= \boxed{.103}$



Find $Z = P_{98}$

98% below — Left Area — .98

2% above — Right Area — .02



2nd VARS ↓

$Z = P_{98} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

left Area, μ, σ

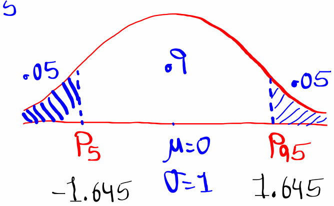
Find two Z-values, round to 3-decimal places, that separate the middle 90% from the rest.

$1 - .9 = .1$

$.1 \div 2 = .05$

$P_5 = \text{invNorm}(.05, 0, 1)$
$= \boxed{-1.645}$

$P_{95} = \text{invNorm}(.95, 0, 1)$
$= \boxed{1.645}$



Normal Prob. dist:

1) Use x, $P(x=c) = 0$

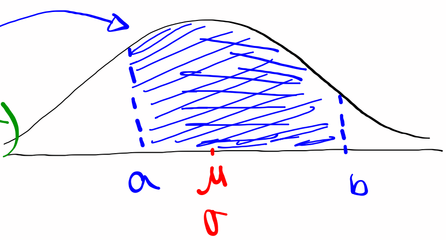
2) Data dist. is symmetric, bell-shape,
   Total area = 1

3) Mean = Mode = Median

4) μ & σ are given in the problem.

5) $\boxed{P(a < x < b)}$ is the corresponding area within the bell-shape graph.

normalcdf(L, U, μ, σ)



$N(\mu, \sigma)$

Given $N(70, 10)$

Normal Prob. dist

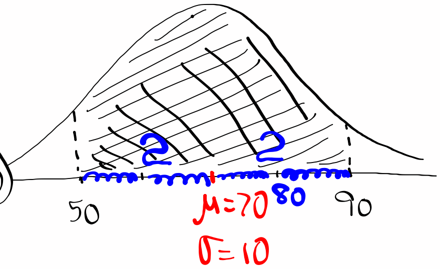
$\mu = 70$

$\sigma = 10$

$P(50 < x < 90)$

$= \text{normalcdf}(50, 90, 70, 10)$

$= \boxed{.954} \approx 95\%$



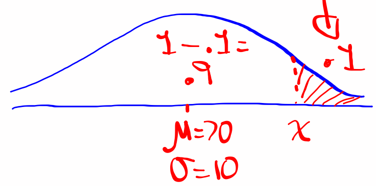
Find x-value, Round to a whole #, that Separates the top 10% from the rest.

Left Area

$x = \text{invNorm}(.9, 70, 10)$

μ σ

$= 82.816 \approx \boxed{83}$



Consider a normal prob. dist with $\mu = 125$, and $\sigma = 15$.

$N(125, 15)$

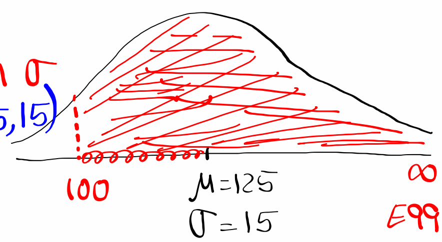
$P(x > 100)$

L U μ σ

$= \text{normalcdf}(100, E99, 125, 15)$

2nd ,

$= \boxed{.952}$



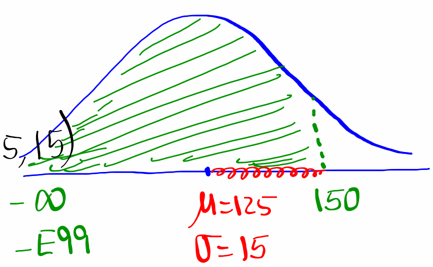
$P(x < 150)$

$= \text{normalcdf}(-E99, 150, 125, 15)$

(−)

2nd ,

$= \boxed{.952}$



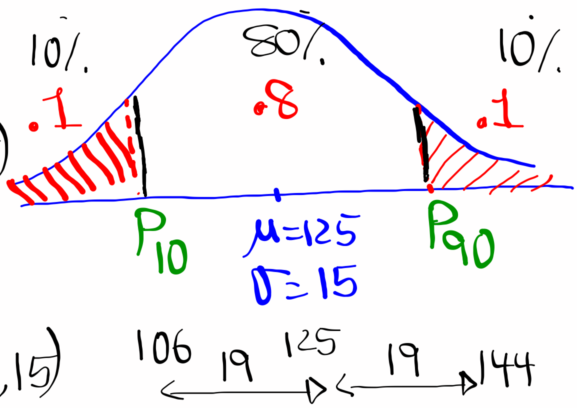
Find two x-values, Round to a whole #, that separate the middle 80% from the rest.

$1 - .8 = .2$

$.2 \div 2 = .1$

$x = P_{10} = \text{invNorm}(.1, 125, 15)$

$\approx \boxed{106}$



$x = P_{90} = \text{invNorm}(.9, 125, 15)$

$\approx \boxed{144}$

Speed of Cars on Certain FWY has a normal dist with $\mu = 68$ mph and $\sigma = 6$ mph. $N(68, 6)$

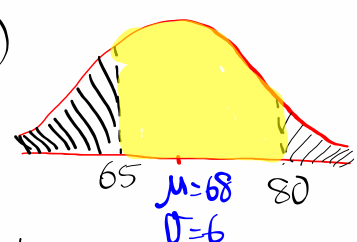
If we randomly Select 1 car,

Find the prob. that speed is below 65 or above 80.

$P(x < 65 \text{ OR } x > 80)$

$= 1 - P(65 < x < 80)$

$= 1 - \text{normalcdf}(65, 80, 68, 6)$



$= \boxed{.331}$

Find K such that $P(x>K)=.01$,

Round to a whole #

Top Area

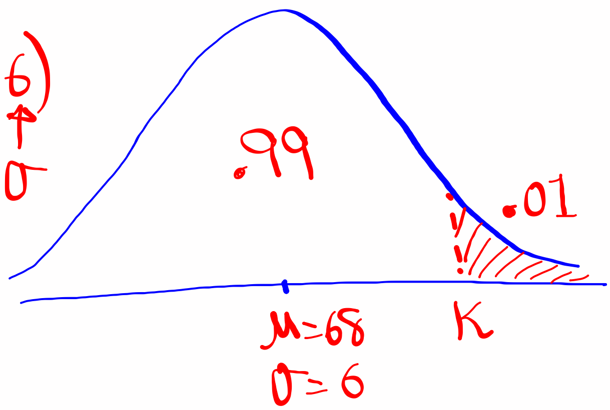
x is the speed of a car on that FWY

$K = \text{invNorm}(.99, 68, 6)$

Left Area

μ

σ



$\approx$ 82 mph

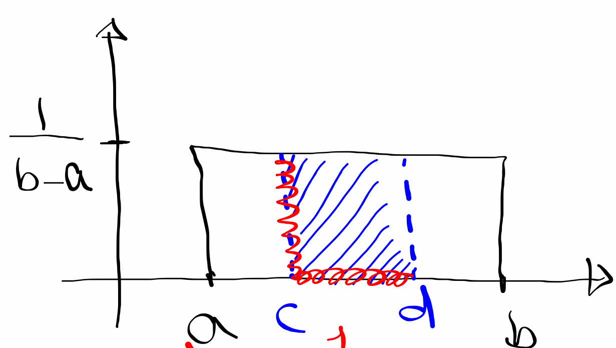
Standard Normal Prob. dist.

Normal prob. dist.

Uniform Prob. dist.

SG 18

Graph is rectangular for all values from a to b.

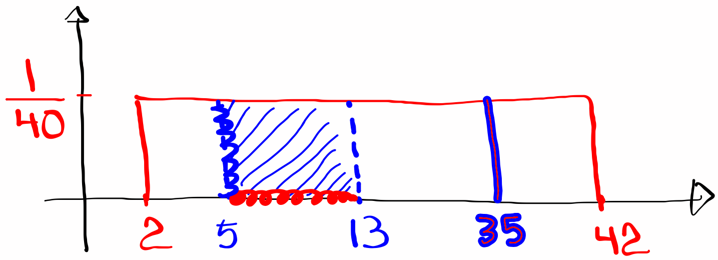


$$P(c<x<d) = (d-c)\cdot\frac{1}{b-a}$$

Consider a Uniform Prob. dist. for all values from 2 to 42.

$P(x=35) = \boxed{0}$

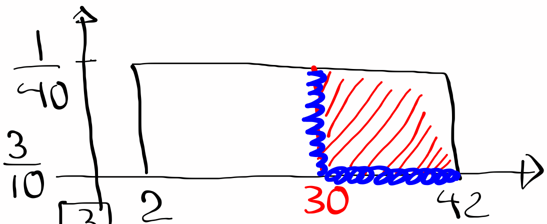
line



$P(5 < x < 13) = (13-5) \cdot \frac{1}{40} = \frac{8}{40} = \frac{1}{5} = \boxed{.2}$

$P(x > 30)$

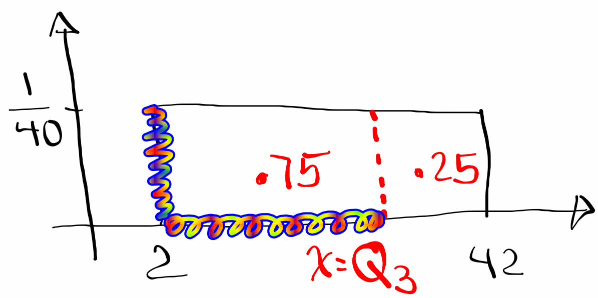
$= (42-30) \cdot \frac{1}{40} = \frac{12}{40} = \frac{3}{10} = \boxed{.3}$



find $x = Q_3$

75% below

25% above



$(x-2) \cdot \frac{1}{40} = .75$

$x - 2 = 40(.75)$

$x - 2 = 30$

$\boxed{x = 32}$ Q_3