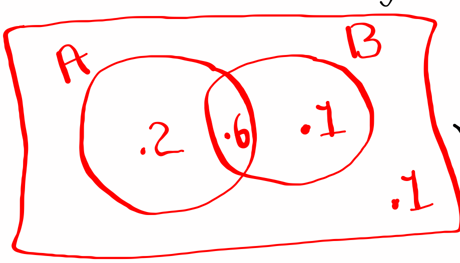
Elementary Statistics Lecture 8

Class QZ 3

Given: $P(A) = .8$, $P(B) = .7$

$P(A \text{ and } B) = .6$ ✓

1) Make Venn Diagram



✓

2) $P(\bar{A})$

$= 1 - .8 = \boxed{.2}$ ✓

3) $P(A \text{ or } B)$

$= .8 + .7 - .6 = \boxed{.9}$

Data
- 1) Qualitative
  - Non-Numerical
- 2) Quantitative
  - Numerical
  - 1) Discrete
    - Countable
  - 2) Continuous
    - Measureable

Let x be a discrete random variable with $\underbrace{\text{prob. dist}}$ $P(x)$.

Prob. dist $P(x)$ gives us the Prob. of all possible outcomes.

Prob. dist. can be in the form of

1) A table

2) A graph

3) A formula

Some rules:

1) $0 \le P(x) \le 1$

2) Sum of all prob. = 1

3) $P(x) = 1 \iff$ Sure event

4) $P(x) = 0 \iff$ Impossible event

5) $0 < P(x) \le .05 \iff$ Rare event

Mean μ "mu" $\mu = \sum x P(x)$

Variance σ^2 "Sigma^2" $\sigma^2 = \sum x^2 P(x) - \mu^2$

Standard deviation σ "Sigma" $\sigma = \sqrt{\sigma^2}$

Consider the table below

x	$P(x)$	$xP(x)$	$x^2P(x)$
1	.2	.2	.2
2	.5	1	2
3	.3	.9	2.7

$\sum xP(x) = 2.1$

$\sum x^2 P(x) = 4.9$

$\mu = \sum x P(x) = \boxed{2.1}$

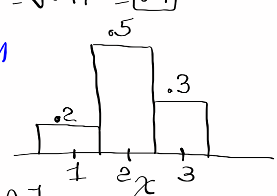
$\sigma^2 = \sum x^2 P(x) - \mu^2$

$= 4.9 - 2.1^2 = \boxed{.49}$

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{.49} = \boxed{.7}$

Draw Prob. dist. histogram



Clear all lists

$x \to L1$

$P(x) \to L2$

1-Var stats L1, L2

list

Freq List

$\mu = \bar{x} = 2.1$

$\sigma = \sigma_x = .7$

$n = 1 \leftarrow$ Total Prob.

Let's find σ^2

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4{:}\ \sigma_x}$ $\boxed{x^2}$ $\boxed{\text{Enter}}$

$\sigma^2 = .49$

Consider the chart below

x	$P(x)$
2	.1
4	.2
6	.3
8	.2
10	.2

1) Find $P(x=10)$

$= 1 - [.1 + .2 + .3 + .2]$

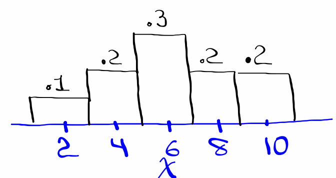
$= 1 - .8 = \boxed{.2}$

2) Find $P(x \geq 4)$

$= 1 - P(x=2)$

$= 1 - .1 = \boxed{.9}$

3) Draw Prob. dist. Histogram



$x \to L1$

$P(x) \to L2$

1-Var Stats L1,L2

$\mu = \bar{x} = 6.4$

$\sigma = \sigma_x = 2.498$

$n = 1$ ✓

Find σ^2 in reduced fraction.

VARS 5: 4: σ x^2

Math 1: ▶Frac Enter

$\sigma^2 = \frac{156}{25}$

A box contains 2 quarters and 3 nickels. Select 2 coins with replacement.

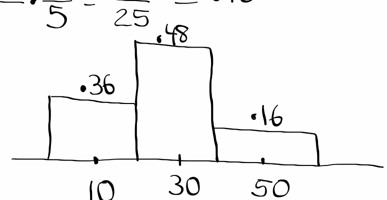
NN	NQ	QN	QQ	Sample Space
↑	↑	↑	↑	
10¢	30¢	30¢	50¢	

$P(10¢) = P(NN) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} = .36$

$P(30¢) = P(NQ \text{ or } QN) = 2 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{25} = .48$

$P(50¢) = P(QQ) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25} = .16$

Total	P(Total)
10¢	.36
30¢	.48
50¢	.16



Total $\to L1$

P(Total) $\to L2$

1-Var Stats L1,L2

$\mu = \bar{x} = 26$

$\sigma = \sigma_x = 13.856$

$n = 1$ ✓

Find σ^2 in reduced fraction

$\boxed{\sigma^2 = 192}$

Application

Expected Value

20 Students bought 1 TKT each for \$10.

A drawing took place.

One tkt drawn.

Winner gets a new calc. worth \$100.

Net gain	P(Net gain)
10 − 100	1/20
10 − 0	19/20

Net gain → L1

P(Net gain) → L2

Expected Value $= \mu = \bar{x}$

1-Var Stats L1, L2

E.V. = \$5

House makes \$5/TKT.

You are taking a flight.

You buy insurance for \$50 for luggage.

Any damages, Airline pays \$500.

Prob. of any damage is .2%

Find expected value per policy sold.

Net gain	P(Net gain)
50 − 500	.2% = .002
50 − 0	1 − .002 = .998

Net gain → L1

P(Net gain) → L2

E.V. $= \mu = \bar{x}$

\$49

Airline makes \$49/policy sold

SG 14 & 15 ✓

Binomial Prob. dist. : SG 16

1) n independent events

2) Each event has only two outcomes.

$P(\text{Success}) = p \qquad P(\text{Failure}) = q$

3) p & q remain unchanged for all events

$p + q = 1 \qquad q = 1 - p$

4) x is # of Successes

$n - x$ is # of failures

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

I randomly selected 10 newborn babies.

Assume having a girl is Success.

$p = .5 \qquad q = .5 \qquad n = 10$

P(exactly 6 girls)

$$P(x=6) = \underset{n}{10}C_{\underset{x}{6}} \cdot (\underset{p}{.5})^{\overset{x}{6}} \cdot (\underset{q}{.5})^{\overset{n-x}{4}} = \boxed{.205}$$

Using TI command:

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ ↓↓ binompdf($\overset{n}{10}$, $\overset{p}{.5}$, $\overset{x}{6}$

Trials: 10 ⬚ ⬚

p: .5 $\boxed{\text{Enter}}$

x-value: 6

$\boxed{\text{Paste}}$ $\boxed{\text{Enter}}$

I tossed a loaded Coin 15 times.

P(land tails) = .6 $n = 15$
$p = .6$ $q = .4$

P(exactly 10 tails)

$= P(x = 10) = \text{binompdf}(15, .6, 10) = \boxed{.186}$

P(at most 10 tails)

$= P(x \le 10) = P(x=10) + P(x=9) + P(x=8) + \cdots + P(x=0)$

$= \text{binomcdf}(15, .6, 10) = \boxed{.783}$

Total Prob.

P(at least 8 tails)

$= P(x \ge 8) = 1 - P(x \le 7)$

we don't want   we want

7 8

$= 1 - \text{binomcdf}(15, .6, 7)$

$= \boxed{.787}$

You are making random guesses on a multiple choice exam with 50 questions. Each question has 4 choices with only one Correct choice.

$n = 50$ $p = \frac{1}{4} = .25$ $q = \frac{3}{4} = .75$

P(exactly 10 Correct Ans) =

$= P(x = 10) = \text{binompdf}(50, .25, 10) = \boxed{.099}$

P(fewer than 15 Correct Ans)

$= P(x < 15) = P(x \le 14) = \text{binomcdf}(50, .25, 14)$

$= \boxed{.748}$

P(more than 10 Correct Ans)

$= P(x > 10) = P(x \ge 11) = 1 - P(x \le 10)$

We don't want   We want

10 11

$= 1 - \text{binomcdf}(50, .25, 10)$

$= \boxed{.738}$

$P(x=a) = \text{binompdf}(n, p, a)$

$P(x \le a) = \text{binomcdf}(n, p, a)$

$P(x \ge a) = 1 - \text{binomcdf}(n, p, a-1)$

$P(a \le x \le b) = \text{binomcdf}(n, p, b) -$
$\text{binomcdf}(n, p, a-1)$

Consider a binomial Prob. dist with
$n = 125$ and $p = .8$

1) $q = 1 - p$
$= \boxed{.2}$

2) $np = 125(.8)$
$= \boxed{100}$

3) $npq =$
$125(.8)(.2) =$
$\boxed{20}$

4) $\sqrt{npq} = \sqrt{20} = \boxed{4.472}$

Assume x is # of Successes,

$P(90 \le x \le 110) = \text{binomcdf}(125, .8, 110) -$
$\text{binomcdf}(125, .8, 89)$
$= \boxed{.982}$

AA says 90% of flights are on time.

I took 40 flights.

1) $n = 40$ 2) $p = .9$ 3) $q = .1$

4) $np = 40(.9) = \boxed{36}$

5) $npq = 40(.9)(.1) = \boxed{3.6}$

6) $\sqrt{npq} = \sqrt{3.6} = \boxed{1.897}$

7) P(between 35 and 39 flights, inclusive, are on time)

$P(35 \le x \le 39) = \text{binomcdf}(40, .9, 39) - \text{binomcdf}(40, .9, 34) = \boxed{.779}$

For Binomial Prob. dist

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard Deviation $\sigma = \sqrt{\sigma^2}$

I flip a fair coin 400 times.

Success is to land tails.

1) $n = 400$ 2) $p = .5$ 3) $q = .5$

4) $\mu = np$
$= 400(.5)$
$= \boxed{200}$

5) $\sigma^2 = npq$
$= 400(.5)(.5)$
$= \boxed{100}$

6) $\sigma = \sqrt{\sigma^2}$
$= \sqrt{100}$
$= \boxed{10}$

7) 68% Range $= \mu \pm \sigma = 200 \pm 10 \Rightarrow \boxed{190 \text{ to } 210}$

8) $\underbrace{\text{Usual Range}}_{\text{95\% Range}} = \mu \pm 2\sigma = 200 \pm 2(10) \Rightarrow \boxed{180 \text{ to } 220}$

9) P(# tails is $\underbrace{\text{between 180 and 220, inclusive}}$)

$P(180 \le X \le 220) = \text{binomcdf}(400, .5, 220) - \text{binomcdf}(400, .5, 179)$

$= \boxed{.960} \approx 96\%$

Exam II

Monday

Details: Read my emails.