Elementary Statistics Lecture 7

Multiplication Rule: (SG 10 - 13)

Keyword: AND

Any event requires multiple Action.

1) Independent events

One outcome does not change the prob. of following events.

Flip a fair coin twice

P(Tails) = .5

P(Heads) = .5

It does not matter the outcome of first flip P(T) = .5, P(H) = .5 on the next flip.

True / false questions

P(correct ans in any question) = .5

Roll a fair die

P(land 6 on each Roll) = $\frac{1}{6}$

Every Roll, Same prob. to land 6.

$P(A \text{ and } B) = P(A) \cdot P(B)$ If $A \& B$ are independent events

A happens, then B happens.

Ex: $P(A) = .3$, $P(B) = .6$, A & B are independent events

1) $P(\bar{A}) = 1 - P(A) = \boxed{.7}$

2) $P(\bar{B}) = 1 - P(B) = \boxed{.4}$

3) $P(A \text{ and } B) = P(A) \cdot P(B) = (.3)(.6) = \boxed{.18}$

4) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .3 + .6 - .18$

$= .9 - .18 = \boxed{.72}$

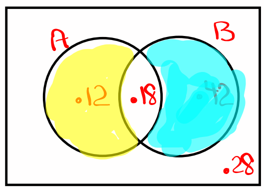
addition Rule

$1 - .72 = .28$

5) Make the Venn Diagram

$P(A \text{ only}) = .3 - .18 = \boxed{.12}$

$P(B \text{ only}) = .6 - .18 = \boxed{.42}$



Total = 1

$P(A \text{only OR } B \text{only})$

$= .12 + .42 = \boxed{.54}$

A full deck of playing cards has 52 cards with 4 Aces.

Draw 2 cards with replacement.

$A \to$ Ace

$\bar{A} \to \overline{\text{Ace}}$

$\underbrace{AA \quad A\bar{A} \quad \bar{A}A \quad \bar{A}\bar{A}}_{\text{Sample Space}}$

$P(AA) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \cdot \frac{1}{13} = \boxed{\frac{1}{169}}$

$P(A\bar{A}) = \frac{4}{52} \cdot \frac{48}{52} = \frac{1}{13} \cdot \frac{12}{13} = \boxed{\frac{12}{169}}$

$P(\bar{A}A) = \frac{48}{52} \cdot \frac{4}{52} = \frac{12}{13} \cdot \frac{1}{13} = \boxed{\frac{12}{169}}$

$P(\bar{A}\bar{A}) = \frac{48}{52} \cdot \frac{48}{52} = \frac{12}{13} \cdot \frac{12}{13} = \boxed{\frac{144}{169}}$

A full deck of playing cards has 52 cards with 12 face cards.

Draw 2 cards without replacement

$F \rightarrow$ face
$\bar{F} \rightarrow \overline{\text{face}}$

$$\Rightarrow \underbrace{FF \quad F\bar{F} \quad \bar{F}F \quad \bar{F}\bar{F}}_{\text{Sample Space}}$$

$$P(FF) = \frac{12}{52} \cdot \frac{11}{51} = \boxed{\frac{11}{221}}$$

$$P(F\bar{F}) = \frac{12}{52} \cdot \frac{40}{51} = \boxed{\frac{40}{221}}$$

$$P(\bar{F}F) = \frac{40}{52} \cdot \frac{12}{51} = \boxed{\frac{40}{221}}$$

$$P(\bar{F}\bar{F}) = \frac{40}{52} \cdot \frac{39}{51} = \boxed{\frac{10}{17}}$$

Dependent Events

Prob. of second draw changes because of first draw

A box has 4 Red, 3 white, and 3 Blue Balls.

Randomly select 3 balls without replacement.

$R \rightarrow$ Red
$W \rightarrow$ white
$B \rightarrow$ Blue

$$P(RRR) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8} = \boxed{\frac{1}{30}}$$

$$P(WWW) = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} = \boxed{\frac{1}{120}}$$

$$P(\text{Red, then White, then Blue}) = \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{3}{8} = \boxed{\frac{1}{20}}$$

Multiplication Rule:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

A happens, then B happens

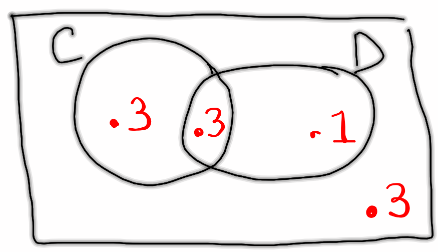
Given

with some algebra

$$\boxed{P(B|A) = \frac{P(A \text{ and } B)}{P(A)}}$$

Conditional Prob.

$P(\text{coffee}) = .6$



$P(\text{Donuts}) = .4$

$P(\text{Coffee and Donuts}) = .3$

$$P(\text{Donuts} \mid \text{Coffee}) = \frac{P(\text{Coffee and Donuts})}{P(\text{coffee})} = \frac{.3}{.6} = \boxed{.5}$$

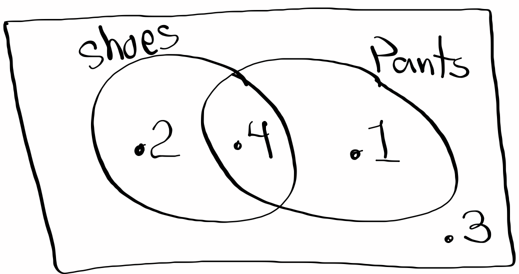
$$P(\text{Coffee} \mid \text{Donuts}) = \frac{P(\text{Coffee and Donuts})}{P(\text{Donuts})} = \frac{.3}{.4} = \boxed{.75}$$

$P(\text{shoes}) = .6$

$P(\text{Pants}) = .5$

$P(\text{shoes} \mid \text{Pants}) = .8$

$P(\text{shoes and pants}) = ?$



Total = 1

$$P(\text{Shoes} \mid \text{Pants}) = \frac{P(\text{Shoes and Pants})}{P(\text{Pants})}$$

$$.8 = \frac{P(S \text{ and } P)}{.5}$$

Cross-Multiply

$P(\text{Shoes and Pants}) = (.8)(.5)$

$P(\text{Shoes and Pants}) = \boxed{.4}$

$P(\text{Shoes or Pants}) =$
$P(\text{Shoes}) + P(\text{Pants}) - P(S \,\&\, P) =$
$.6 + .5 - .4 = \boxed{.7}$

I have 5 people, I need to select 2 of them.

Adam Bill Carol David Eric

~~AB~~	~~AC~~	~~AD~~	~~AE~~
BA	~~BC~~	~~BD~~	~~BE~~
CA	CB	~~CD~~	~~CE~~
DA	DB	DC	~~DE~~
EA	EB	EC	ED

20 Selections

what if oder does not matter

10 Selections

$_nC_r$ Combination

$_5C_2$ 10

5 [Math] [→] PRB [↓] [$_nC_r$] 2 Enter

12 Basketball players on a team

Coach needs 5 players to start the game.

How many Selections?

$${}_{12}C_5 = 792$$

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6 Males & 4 Females

Need to select 3 people.

$$P(\text{All Females}) = \frac{{}_4C_3}{{}_{10}C_3} = \frac{4}{120} = \frac{1}{30}$$

$$P(\text{All Males}) = \frac{{}_6C_3}{{}_{10}C_3} = \frac{20}{120} = \boxed{\frac{1}{6}}$$

$$P(2F \,\&\, 1M) = \frac{{}_4C_2 \cdot {}_6C_1}{{}_{10}C_3} = \frac{36}{120} = \frac{6}{20} = \boxed{\frac{3}{10}}$$

$$P(1F \,\&\, 2M) = \frac{{}_4C_1 \cdot {}_6C_2}{{}_{10}C_3} = \frac{60}{120} = \boxed{\frac{1}{2}}$$

A full deck of playing Cards has 52 Cards, 12 Face Cards, 4 Aces.

Draw 5 Cards

other Cards

$$P(2 \text{ Face Cards } \& \ 2 \text{ Aces}) = \frac{{}_{12}C_2 \cdot {}_4C_2 \cdot {}_{36}C_1}{{}_{52}C_5}$$

$$= \frac{14256}{2598960} = .005$$

Prob. with at least 1:

8 Males, 4 Females, Select 3 people

FFF

Some F & Some M

MMM

$P(\text{All Females}) = \frac{4}{12} \cdot \frac{3}{11} \cdot \frac{2}{10} = \boxed{\frac{1}{55}}$

$P(\text{All Males}) = \frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10} = \boxed{\frac{14}{55}}$

$P(\text{at least 1 Female})$

Total Prob.

$= 1 - P(\text{No Females}) = 1 - \frac{14}{55} = \boxed{\frac{41}{55}}$

$P(\text{at least 1 Male})$

$= 1 - P(\text{All Females})$

$= 1 - \frac{1}{55} = \boxed{\frac{54}{55}}$

FFF

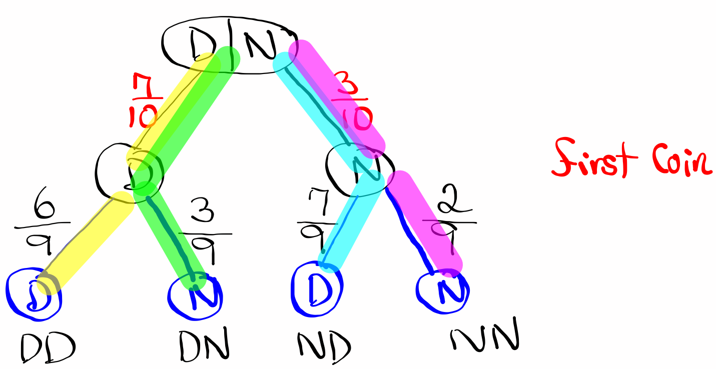
Some F & Some M

MMM

Multiplication Rule with Tree diagram:

7 Dimes & 3 Nickels

Select 2 Coins, No replacement



$P(DD) = \frac{7}{10} \cdot \frac{6}{9} = \boxed{\frac{42}{90}}$

$P(ND) = \frac{3}{10} \cdot \frac{7}{9} = \boxed{\frac{21}{90}}$

$P(DN) = \frac{7}{10} \cdot \frac{3}{9} = \boxed{\frac{21}{90}}$

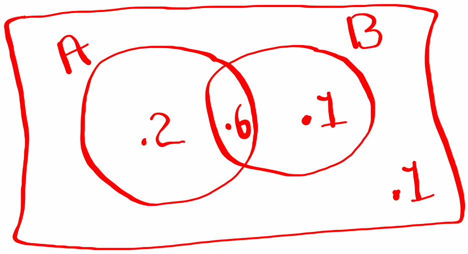
$P(NN) = \frac{3}{10} \cdot \frac{2}{9} = \boxed{\frac{6}{90}}$

Class QZ 3

Given: $P(A) = .8$, $P(B) = .7$

$P(A \text{ and } B) = .6$

1) Make Venn Diagram



2) $P(\bar{A})$

$= 1 - .8 = \boxed{.2}$

3) $P(A \text{ or } B)$

$= .8 + .7 - .6 = \boxed{.9}$