Elementary Statistics Lecture 4

Consider the Sample below

85	72	65	80	90
93	55	60	100	91
83	77	75	98	100
68	72	82	80	85

Clear all lists
[2nd] [+] [4:] [Enter]

Store this Sample in L1

[STAT] Edit
[1: Edit]

L1
85
72
65
⋮
85

Find $\bar{x}$ and S. Round to a (whole #)

[STAT] [CALC]
[1: 1-Var Stats]

$\bar{x} = 80.55$
$S = S_x = 12.829$

$\tilde{x} = 81$
$S = 13$

Find S^2 in reduced fraction

[VARS] [5: Statistics] [3: S_x] [x^2] [MATH] [1: ▶Frac] [Enter]

$$S^2 = \frac{62539}{380}$$

Find

68% Range ⟹ $\bar{x} \pm S = 81 \pm 13$ ⟹ [68 to 94]

Usual Range ⟹ $\bar{x} \pm 2S = 81 \pm 2(13)$ ⟹ [55 to 107]
95% Range

Let's find the 5-Number Summary

STAT → CALC

1: 1-Var Stats

Once we see $\bar{x}, \Sigma x, \ldots$

↓ Min = 55

↓ Q₁ = 72

↓ Med = 81

↓ Q₃ = 90.5

Max = 100

[Box plot with values: 55, 72, 81, 90.5, 100]

$IQR = Q_3 - Q_1 = 90.5 - 72 = \boxed{18.5}$

Upper Fence $= Q_3 + 1.5(IQR)$
$= 90.5 + 1.5(18.5) = 118.25$

Lower Fence $= Q_1 - 1.5(IQR)$
$= 72 - 1.5(18.5) = 44.25$

There is no data element below LF or above UF ⇒ **No Outliers**

Let's Sort L1, and make STEM Plot

STAT Edit

2: SortA(2nd 1 Enter

```
5 | 5
6 | 0 5 8
7 | 2 2 5 7
8 | 0 0 2 3 5 5
9 | 0 1 3 8
10 | 0 0
```

Let's view L1

2nd 1 Enter

1) How many data elements are below 75? 6

2) What percent of data elements are below 75?

$\frac{6}{20} \cdot 100 = \boxed{30\%}$

Percentile ↳ $P_{30} = 75$

30% below 75   70% above 75.

$\underline{\quad 30\% \quad | \quad 70\% \quad}$
below it 75   above it.

A data set has a mean of 120 and Stand. dev. of 15.

1) find the Z-Score for data element 165.

$$Z = \frac{x - \bar{x}}{S} = \frac{165 - 120}{15} = \frac{45}{15} = 3$$

165 is an unusual data element

$$Z < -2 \quad OR \quad Z > 2$$

2) find a data element with Z-Score of −1.8.

$$Z = \frac{x - \bar{x}}{S}$$

$$-1.8 = \frac{x - 120}{15}$$

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Cross-Multiply, Solve for x.

$$x - 120 = -1.8(15) \qquad x = 120 - 27$$

$$x - 120 = -27 \qquad \boxed{x = 93}$$

What is standard deviation?

$$S = \sqrt{S^2}$$

Standard deviation is non-negative.

It is a numerical value that indicates how data elements deviate (Vary) from the mean.

When S is Small ⇒ Data elements are close to the mean.

When S is large ⇒ data elements are more spread out from the mean.

when S = 0 ⇒ All data elements are the same and = to mean

Clear all lists

Store the following in L1

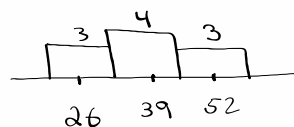| 10 | 10 | 10 | 10 | $\Rightarrow$ | $\bar{x} = 10$ |
| | | | | | $S = 0$ |

| 10 | 10 | 10 | 10 |

Change the first 10 to 11 $\Rightarrow$ $\bar{x} = 10.125$
$S = .354$

Now change 11 to 1000 $\Rightarrow$ $\bar{x} = 133.75$
$S = 350.018$

Consider the chart below

class limits	Class MP	Class F
20 — 32	26	3
33 — 45	39	4
46 — 58	52	3

(+cw
(+13

Find
$\bar{x} = 39$
$S = 10.614$

Change 4 to 40
Find
$\bar{x} = 39$
$S = 4.750$

Class MP → L1 ← list
Class F → L2 ← Freq List

Now change 40 to 1 Find $\bar{x} = 39$

$S = 13$
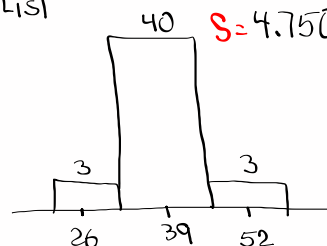
Percentile

"Data must be Sorted"

$P_{10} \to 10\%$ below $\& 90\%$ above

$$\frac{\overset{10\%}{\rule{0pt}{0pt}} \qquad\qquad \overset{90\%}{\rule{0pt}{0pt}}}{\underset{P_{10}}{\rule{0pt}{0pt}}}$$

$P_{80} \to 80\%$ below $\& 20\%$ above

$$\frac{\overset{80\%}{\rule{0pt}{0pt}} \qquad\qquad \overset{20\%}{\rule{0pt}{0pt}}}{\underset{P_{80}}{\rule{0pt}{0pt}}}$$

P_K

$$\frac{\overset{K\%}{\rule{0pt}{0pt}} \qquad\qquad \overset{(100-K)\%}{\rule{0pt}{0pt}}}{\underset{P_K}{\rule{0pt}{0pt}}}$$
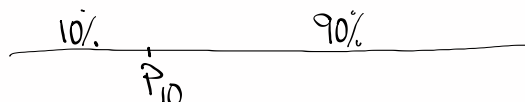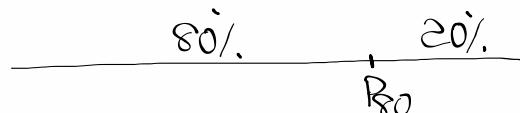
How to find P_K:

$$L = \frac{K}{100} \cdot n$$

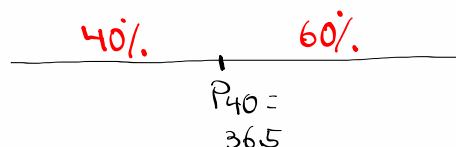If L is decimal $\Rightarrow$ Round-up $\to P_K = L$th element

If L is a whole $\Rightarrow P_K = \dfrac{L\text{th element} + \text{Next one}}{2}$
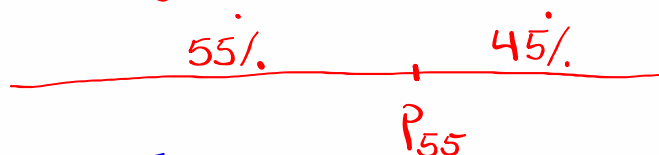
$n = 25$

Find P_{40}

$L = \dfrac{40}{100} \cdot 25 = 10$

$P_{40} = \dfrac{10\text{th element} + \text{Next one}}{2}$

$= \dfrac{35 + 38}{2}$

$\boxed{P_{40} = 36.5}$

1	89
2	0137
3	025589
4	03555689
5	2348
6	5

$$\frac{\overset{40\%}{\rule{0pt}{0pt}} \qquad\qquad \overset{60\%}{\rule{0pt}{0pt}}}{\underset{\begin{array}{c}P_{40} = \\ 36.5\end{array}}{\rule{0pt}{0pt}}}$$

find P_{55}

55% | 45%

P_{55}

$L = \dfrac{55}{100} \cdot 25 = 13.75$

$L = 14$ $P_{55} = $ 14th element $\boxed{P_{55} = 43}$

```
1 | 89
2 | 0137
3 | 025589
4 | 03555689
5 | 2348
6 | 5
```

doing Reverse:

find K such that $P_K = 50$

Below

$\dfrac{K\%}{} \quad \dfrac{(100-K)\%}{}$

50

$K = \dfrac{B}{n} \cdot 100 = \dfrac{20}{25} \cdot 100 = 80 \;\rightarrow\; \boxed{P_{80} = 50}$

Sample Size
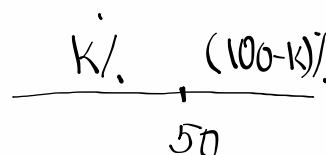
Scores of 32 exams are given below

```
5  | 268
6  | 03557
7  | 025558889
8  | 2366889
9  | 0225789
10 | 0
```

$n = 32$

find P_{10}

$L = \dfrac{10}{100} \cdot 32 = 3.2 \;\rightarrow\; L = 4$

$P_{10} = $ 4th element $= \boxed{60}$

find P_{50}

$L = \dfrac{50}{100} \cdot 32 = 16$

$P_{50} = \dfrac{16\text{th element} + \text{Next element}}{2}$
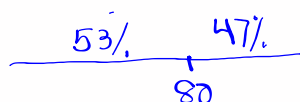
$= \dfrac{78+79}{2} = \boxed{78.5}$

Median

find percentile
ranking for Score 80.
find K such that $P_K = 80$

≈ 53

$K = \dfrac{B}{n} \cdot 100$

$= \dfrac{17}{32} \cdot 100$

$= 53.125$

53% | 47% $\boxed{P_{53} = 80}$

80

Ages of 24 randomly selected students are given below

28	25	32	40	18
20	19	35	25	45
50	20	17	23	38
48	33	25	25	39
40	52	18	20	

Sort, Make STEM Plot

```
1 | 7889
2 | 000355558
3 | 23589
4 | 0058
5 | 0 2
```

find P_{20}

$L = \frac{20}{100} \cdot 24 = 4.8 \rightarrow L = 5$

$P_{20} = $ 5th element $= 20$

find P_{80}

$L = \frac{80}{100} \cdot 24 = 19.2 \rightarrow L = 20$

$P_{80} = $ 20th element $= 40$

find P_{50} ← Median

$L = \frac{50}{100} \cdot 24 = 12$

$P_{50} = \frac{12\text{th} + \text{Next}}{2} = \frac{25 + 28}{2} = \boxed{26.5}$

---

find K Such that

$P_K = 35$ } Below

$K = PR = \frac{B}{n} \cdot 100 = \frac{15}{24} \cdot 100$

↑ Sample Size

```
1 | 7889
2 | 000355558
3 | 23589
4 | 0058
5 | 0 2
```

$= 62.5 \rightarrow 63\%$

$\boxed{P_{63} = 35}$

find P_{100} undefined

← highest percentile is 99.

Complete the chart below

x (L1)	y (L2)	x^2	y^2	xy
1	2	1	4	2
2	5	4	25	10
3	4	9	16	12
4	6	16	36	24

$\sum x = 10$ $\sum y = 17$

$\sum x^2 = 30$ $\sum y^2 = 81$

$n = 4$ $\sum xy = 48$

Clear all lists

$x \to L1$

$y \to L2$

[STAT] [→] CALC

[2:2-VarStat]

xlist: L1

Ylist: L2

FreqList: blank

[Calculate]

L1, L2

[Enter]

Plot the following Points

$(2,3)$, $(3,6)$, $(4,4)$, $(6,8)$

Scatter Plot

Regression line

$y = 1.286 + 1.057x$ $y = a + bx$

$x \to L1$

$y \to L2$
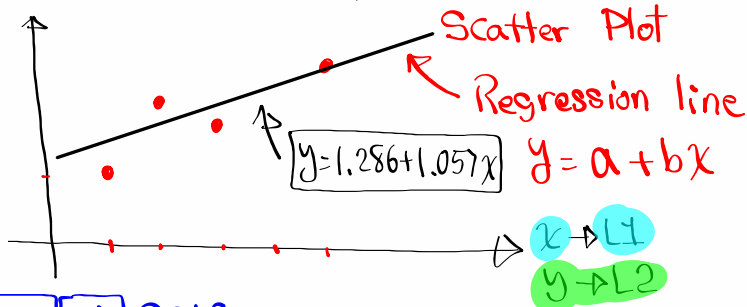
[STAT] [→] CALC

[8: LinReg(a+bx)]

No Menu   L1, L2 enter

$a = 1.286$

$b = 1.057$

xlist: L1

Ylist: L2

clear

[Calculate]

Study time | QZ Score

Study time	QZ Score
2	8
3	9
2	10
1	7
0	5
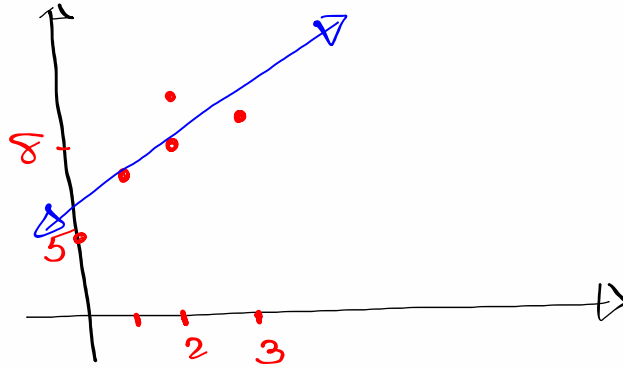
Study time $\rightarrow x \rightarrow L1$

QZ Score $\rightarrow y \rightarrow L2$

$a \approx 5$

$b = 1$ $\rightarrow \boxed{y = 5 + x}$

STAT CALC

8: LinReg(a+bx)

$a = 5.462$

$b = 1.462$

Class QZ 1:

Consider the Sample below

8   12   10   10   7

15   13   20   18   12

Find

1) $\bar{x}$

$\bar{x} = 12.5$

$S = 4.170$

$S^2 = \dfrac{313}{18}$

2) S in 3-decimals.

3) S^2 in reduced fraction