Elementary Statistics Lecture 12

Testing claims SG 24-27

Claim could be made about any parameters.

- Population Proportion P
- Population Mean μ
- Population standard deviation σ

Purpose of testing:

It is to determine the validity of a claim using some significance level.

Final Conclusion:

Reject the claim OR Fail-to-Reject the claim (Support)

Common Sense:

when claim is valid ⇒ we fail-to-reject the claim.

when claim is invalid ⇒ we reject the claim.

Possible errors:

we reject a valid claim.

we fail-to-reject an invalid claim.

Final Conclusion:

Reject the claim (when claim is invalid)

OR

Fail-to-Reject the claim (when claim is valid)

Testing Methods:

- Traditional Method
- P-value Method

we use these two methods.

- Confidence Interval Method

Regardless of method used, final conclusion must be the same.

Final Conclusion:

Reject the claim OR FTR the claim

Look at the left side of study guides 24-27 on my website, get a copy of Testing Chart.

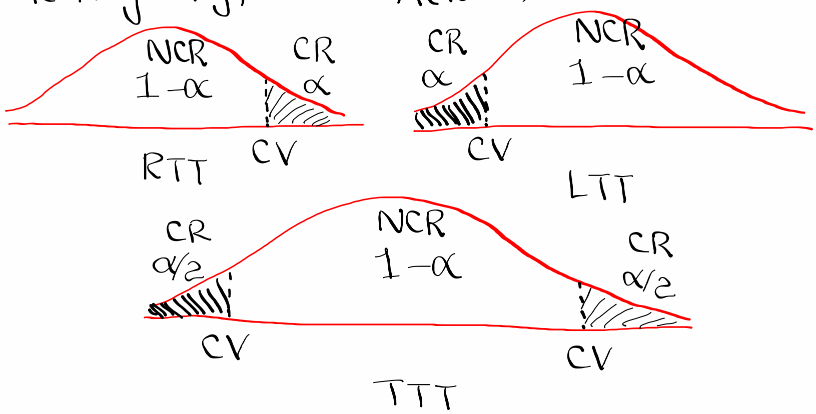
Testing Types:

1) Right-Tail-Test
2) Left-Tail-Test
3) Two-Tail-Test

Every testing comes with Significance level α. $0<\alpha<1$

when α not given $\Rightarrow$ use .05

Testing types in Pictures:



Testing Process:

1) Set up H_0 & H_1 — Null Hypothesis, Alternative Hypothesis

H_a

H_0 most contain = Sign.

$=, \geq, \leq$

H_1 cannot have = Sign.

$\neq, <, >$

Keywords:

H_0: is, equal, Same, at least, at most, ...

H_1: isnot, not equal, different, more than, less than, above, below, exceed, ...

Claim could be H_0 or H_1 but not both at the Same time.

Always identify the claim & Type of testing.

2) Find all Critical values.

Drawing, labeling, shading and Full TI command required.

3) Find Computed Test statistic (CTS), and P-value.

Full TI command or Formula required.

4) use the testing Chart to determine the Validity of H_0 & H_1

H_0 Valid $\Leftrightarrow$ H_1 invalid

H_0 invalid $\Leftrightarrow$ H_1 Valid

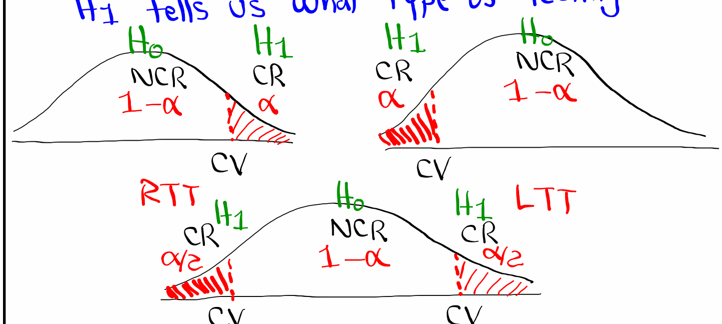
5) Final Conclusion must be about the Claim

Reject the claim OR FTR the claim

More on H_0 & H_1:

$H_0: =$	$H_0: \geq$	$H_0: \leq$
$H_1: \neq$	$H_1: <$	$H_1: >$
TTT	LTT	RTT

H_1 tells us what type of Testing to do.



TTT

$$\boxed{\begin{array}{l} P(H_0 \text{ Valid}) = 1-\alpha = P(H_1 \text{ invalid}) \\ P(H_0 \text{ invalid}) = \alpha = P(H_1 \text{ Valid}) \end{array}}$$

I claim that 10% of all Students Smoke.

$P = .1$

Equal Sign $\Rightarrow H_0$

$H_0: P = .1$ claim

$H_1: P \neq .1$ TTT

I claim the mean age of all College Students is at most 30 Yrs.

$\mu \leq 30$

with = $\Rightarrow H_0$

$H_0: \mu \leq 30$ claim

$H_1: \mu > 30$ RTT

I claim that standard deviation of Salaries of all nurses is below \$400.

$\sigma < 400$

NO = Sign $\Rightarrow H_1$

$H_0: \sigma \geq 400$

$H_1: \sigma < 400$ claim, LTT

Campus bookstore claims that the mean Price of all textbooks is not \$100.

$\underbrace{\mu \neq 100}_{H_1}$

$H_0: \mu = 100$

$H_1: \mu \neq 100$ claim, TTT

I claim that Stand. dev. of all math exams exceeds 10.

$\sigma > 10$ H_1

$H_0: \sigma \leq 10$

$H_1: \sigma > 10$ claim, RTT

Four-Possible outcomes for H_0:

Conclusion \ Reality	H_0 Valid	H_0 invalid
Support H_0	Correct Decision	Type II error
Reject H_0	Type I error	Correct Decision

$P(H_0 \text{ Valid}) = 1 - \alpha = P(H_1 \text{ invalid})$

$P(H_0 \text{ invalid}) = \alpha = P(H_1 \text{ Valid})$

I claim that 70% of all students love online classes.

$H_0: p = .7$ claim

$H_1: p \neq .7$ TTT

Assume H_0 is valid

Describe Type I error.

If we reject that $\boxed{\text{70\% of all students love online classes}}$

H_0

CNN claims that the mean age of all voters in the last election was at most 55.

$\mu \leq 55$

$H_0: \mu \leq 55$ claim

$H_1: \mu > 55$ RTT

Assume H_0 is invalid

Describe $\boxed{\text{type II error}}$

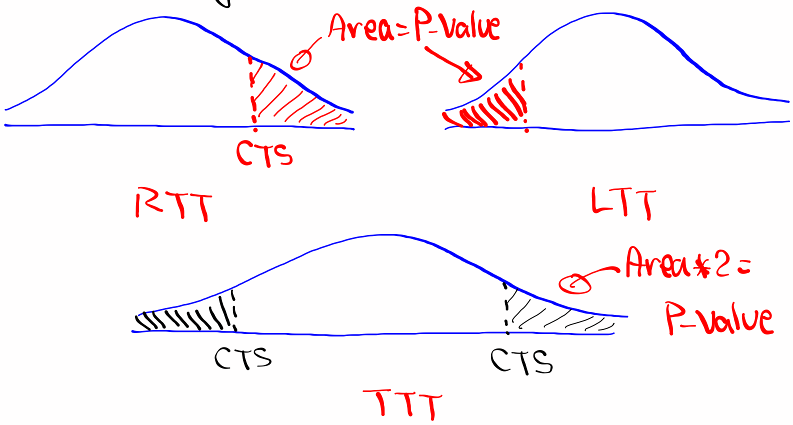
I support that the $\boxed{\text{mean age of all voters was at most 55.}}$

H_0

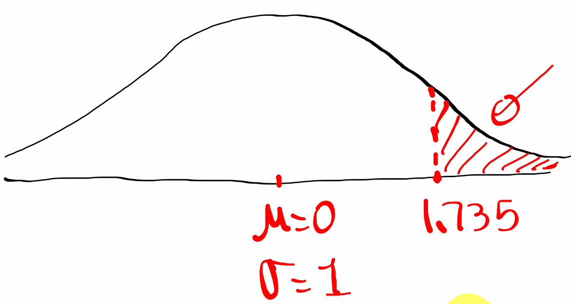
what is P-Value?

P-Value is the area of the tail marked by CTS. (Computed Test Statistic)

Multiply that area by 2 only when Performing TTT.



CTS $Z=1.735$, RTT, find p-value.

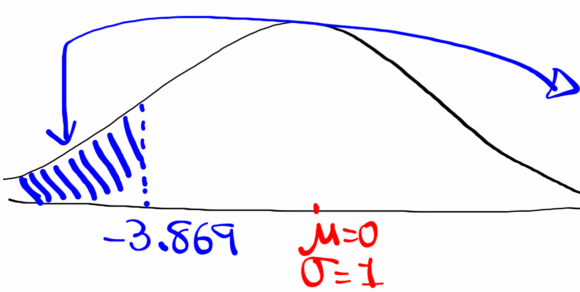


Area = P-Value

= normalcdf(1.735, E99, 0, 1)

= $\boxed{.041}$

CTS $Z=-3.869$ TTT, find P-Value

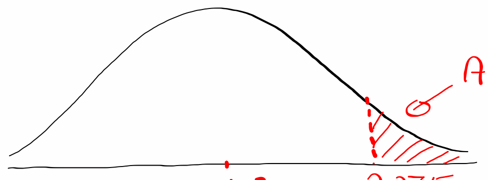


Area * 2 = P-Value

2 * normalcdf(-E99, -3.869, 0, 1)

= $\boxed{1.09 \times 10^{-4}} \approx \boxed{1.1 \times 10^{-4}}$

CTS $t=2.375$, $df=17$, TTT, find P-value.



Area * 2 = P-value

$2 * tcdf(L, U, df)$

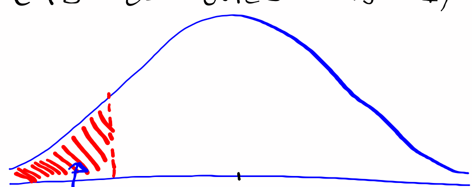
σ unknown

$df=17$

$2 * tcdf(2.375, E99, 17)$

$= \boxed{.030}$

CTS $t=-8.125$ $df=11$, LTT find P-value



0.0000028

σ unknown

$df=11$

$tcdf(-E99, -8.125, 11) = \boxed{2.8\times10^{-6}}$

Determine min. Sample Size:

1) Population Proportion

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} \xrightarrow{\text{with}} \text{Some Algebra} \quad n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

when decimal Round-up.

If $\hat{p}$ and $\hat{q}$ are both unknown ⇒ use .5 for each.

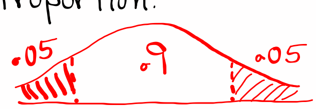
Suppose $\hat{p}=.3$ C-level: .9, $E=4\%$

Find min. Sample Size needed to construct Conf. interval for Pop. Proportion.

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= (.3)(.7)\left(\frac{1.645}{.04}\right)^2$$

$$= 355.16\ldots \quad \boxed{n = 356}$$

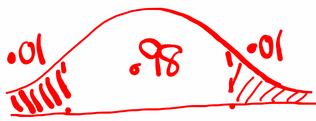


$Z_{.05} = invNorm(.95, 0, 1)$

$= \boxed{1.645}$

Find min. Sample Size needed to construct 98% conf. interval for pop. proportion and error not to exceed 5%.

1) $\hat{p} = .4$



$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$Z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

$$= (.4)(.6)\left(\frac{2.326}{.05}\right)^2$$

$= 519.38\ldots$

$\boxed{n = 520}$

2) $\hat{p}$ & $\hat{q}$ are unknown.

use .5 for each

$$n = (.5)(.5)\left(\frac{2.326}{.05}\right)^2$$

$= 541.0276$

$\boxed{n = 542}$

Determine min. Sample Size:

2) Population mean

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \xrightarrow{\text{with}} \text{Some Algebra}$$

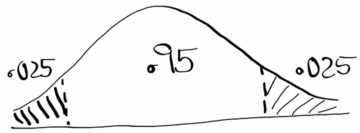
$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

if decimal ⇒ Round-up

If σ is unknown ⇒ use S instead.

Find min. Sample Size needed to construct 95% Conf. interval for pop. mean and error not to exceed 10.

1) $\sigma = 25$ $\quad n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1.960 \cdot 25}{10}\right)^2$



$= 24.01 \Rightarrow n = 25$

$z_{.025} =$ invNorm(.975, 0, 1) = $\boxed{1.960}$

2) Suppose $S = 30$, σ unknown, and new $E = 5$.

$n = \left(\frac{1.960 \cdot 30}{5}\right)^2 = 138.29\ldots \quad \boxed{n = 139}$