Elementary Statistics Lecture 11

Estimating Parameters

SG 22&23

α Alpha

$0 < \alpha < 1$ It is called Significance level.

When α not given $\Rightarrow$ use .05

$\alpha/2$ is the area of right side of the graph of Prob. dist.

$(1-\alpha)\cdot 100\%$ is called Confidence level.

α	Conf. level
.01	$(1-.01)\cdot 100\% = 99\%$
.02	$(1-.02)\cdot 100\% = 98\%$
.05	$(1-.05)\cdot 100\% = 95\%$ Usual Range

$\alpha/2$ is the area of the right tail.

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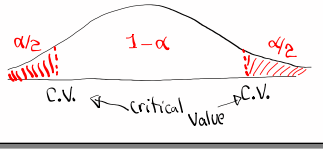
$(1-\alpha)$ is the middle Area



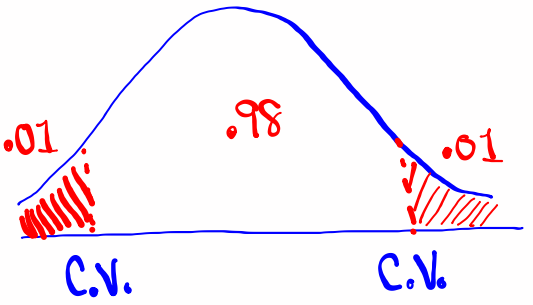
Suppose $\alpha = .02$

$\alpha/2 = .02/2 = .01$

$1 - \alpha = 1 - .02 = .98$

Confidence level: 98%

Significance level: .02



Now

$Z_{\alpha/2}$ ← It is the area of the right Tail.

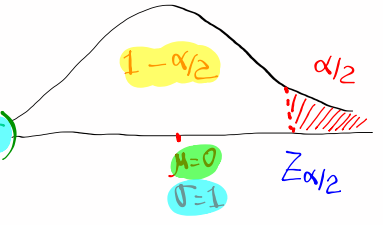
Standard Normal Prob. dist

Symmetric, Bell-Shape, Total Area = 1

$\mu = 0$, $\sigma = 1$

How to find $Z_{\alpha/2}$:

invNorm(Left Area, μ, σ)

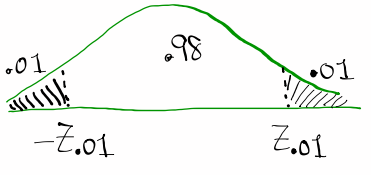


Find $Z_{.01}$

$\frac{\alpha}{2} = .01$

$\alpha = .02$

$1 - \alpha = .98 = 98\%$



$Z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

Find $\pm Z_{\alpha/2}$ for 90% Confidence level.

Middle Area .9

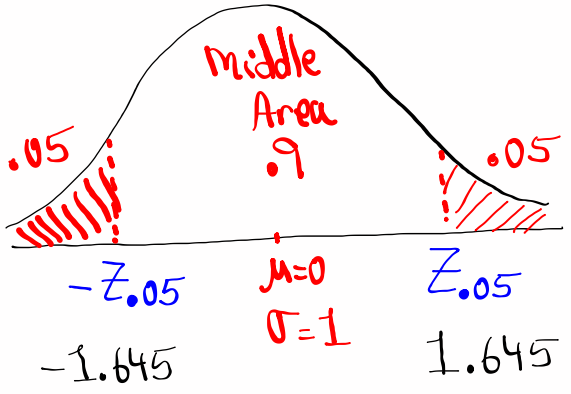
$1 - .9 = .1 = \alpha$

$\alpha/2 = .1/2 = .05$ ← Area of each tail

$Z_{.05} =$

invNorm(.95, 0, 1)

$= \boxed{1.645}$



Sample ⟷ Statistic

Population ⟷ Parameters

To estimate parameters, we begin with Statistic.

To estimate	we Start with
Population proportion P	Sample Proportion $\hat{p}$ (P-hat)
Population Mean μ	Sample Mean $\bar{x}$ (x-bar)
Population Standard deviation σ	Sample Standard deviation s

→ Point-estimate

Our estimation of any Parameter comes as range of values.

These range of values are Called Confidence Interval.

Every Confidence Interval comes with Confidence level (C-level).

Some famous C-level:

90%, 95%, 98%, 99%

If C-level is not given $\Rightarrow$ use 95%

Final Answer for Confidence Interval for

Population Proportion	$\rightarrow$	$<p<$
" Mean	$\rightarrow$	$<\mu<$
" Standard deviation	$\rightarrow$	$<\sigma<$

Constructing Confidence Interval for Population Proportion p:

Final Answer: $<p<$

Format: $\hat{p} - E < p < \hat{p} + E$

$\hat{p}$ Sample Proportion "Point-estimate"

Margin of error

$$\hat{p} = \frac{x}{n}$$

x ← # of favorable responses

n ← Sample Size

$$\hat{q} = 1 - \hat{p}$$

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$z_{\alpha/2}$ is the Critical Value associated with $(1-\alpha)\cdot 100\%$ C-level.

I Surveyed 250 high School Students, and 75 of them were Smokers.

$n=250$ $\hat{p}=\frac{x}{n}=\frac{75}{250}=.3$ $\hat{p}=.3$ $\hat{q}=1-\hat{p}=.7$

$x=75$

Find 98% Confidence Interval for the prop. of all high School Students that Smoke.

C-level: .98

$E=z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$=2.326\cdot\sqrt{\frac{(.3)(.7)}{250}}$

$E=.067$

$z_{.01}=\text{invNorm}(.99,0,1)=2.326$

$\hat{p}-E<p<\hat{p}+E$

I am 98% Confident that between 23% and 37% of all high School Students Smoke.

$.3-.067<p<.3+.067$

$.233<p<.367$

$.233<p<.367$

STAT
TESTS
1-PropZInt
x:75
n:250
C-level:.98
Calculate

$\hat{p}=\frac{.367+.233}{2}=.3$

$E=\frac{.361-.233}{2}=.067$

I Surveyed 400 College Students and 6% of them were left-handed.

$n=400$

$\hat{p}=.06$

$\hat{q}=.94$

$x=n\hat{p}$

$x=400(.06)\Rightarrow x=24$

if decimal → Round-up

Construct 90% Conf. interval for the proportion of all students that are left-handed.

1-Prop ZInt
x:24
n:400
C-level:.9
Calculate

$.040<p<.080$

$\hat{p}=\frac{.08+.04}{2}=\frac{.12}{2}=.06$

$E=\frac{.08-.04}{2}=\frac{.04}{2}=.02$

I Surveyed 585 LA residents, and 90% of them were fan of Lakers.

$n = 585$

$\hat{p} = .9$

$\hat{q} = .1$

$x = n\hat{p} = 585(.9) = 526.5$ if decimal $\Rightarrow$ Round-up $\quad \boxed{x = 527}$

Find 99% Confidence Interval for the prop. of all LA residents that are fan of Lakers.

C-level: .99

1-PropZInt

$\boxed{.869 < p < .933}$

$E = \dfrac{.933 - .869}{2} = \boxed{.032}$

$\hat{p} = \dfrac{.933 + .869}{2} = \boxed{.901}$

t-Dist:

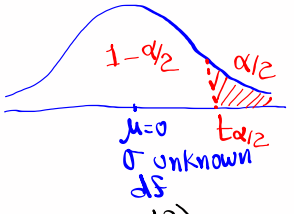
Symmetric, Bell-Shape, total area = 1

$\mu = 0$, σ unknown

It comes with degrees of freedom df.

$t_{\alpha/2}$ ← Right-Tail Area

↑ Critical Value



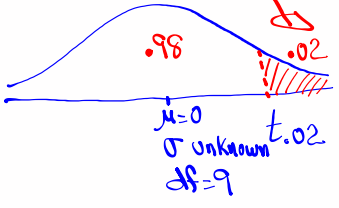
How to find $t_{\alpha/2}$

2nd VARS ↓ invT(Left Area, df)

find $t_{.02}$ with df = 9.

$t_{.02} = \text{invT}(.98, 9)$

$= \boxed{2.398}$

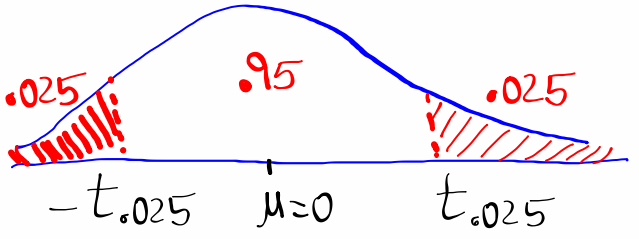


Find $\pm t_{\alpha/2}$ for 95% C-level with df=14.

Middle Area
.95

$1 - .95 = .05$

$.05 \div 2 = .025$



$t_{.025} = \text{invT}(.975, 14)$

$= \boxed{2.145}$

Find $t_{\alpha/2}$ for $\alpha = .01$ with df=19.

$1 - \alpha = .99$

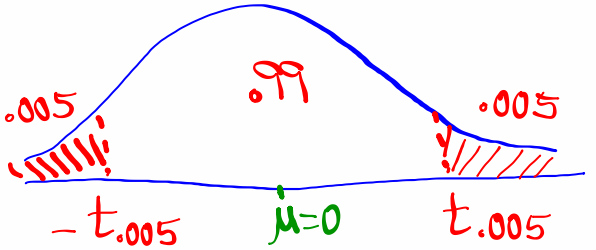
C-level: 99%

Middle Area = .99

$\alpha/2 = .01 \div 2 = .005$

Left Tail Area

Right tail Area



$t_{.005} = \text{invT}(.995, 19)$

$= \boxed{2.861}$

Constructing Confidence Interval for pop. mean μ:

Answer $\quad < \mu <$ (Sample Mean "Point-estimate")

Format $\quad \bar{x} - E < \mu < \bar{x} + E$ (E: Margin of error)

Case I: σ Known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $\rightarrow df = n-1$
STAT TESTS ZInterval inpt: Stats	STAT TESTS TInterval inpt: Stats

$\bar{x} = \frac{+}{2}$, $E = \frac{-}{2}$

Given : $n = 28$, $\bar{x} = 75$, $\sigma = 12$

Find 90% Conf. Interval for the pop. mean.

σ Known $\Rightarrow$ ZInterval

inpt: Stats

$\sigma = 12$

$\bar{x} = 75$

$n = 28$

C-level: .9

$71 < \mu < 79$

$\bar{x} = \frac{79+71}{2} = 75$

$E = \frac{79-71}{2} = 4$

Given: $n = 12$ $\bar{x} = 85$ $s = 10$

Find 90% conf. interval for pop. mean.

σ unknown $\Rightarrow$ T Interval

inpt: Stats

$80 < \mu < 90$

$\bar{x} = \frac{90+80}{2} = 85$

$E = \frac{90-80}{2} = 5$

I randomly Selected 40 nurses, their mean monthly Salary was \$6200.

$n = 40$

$\bar{x} = 6200$

It is known that Standard deviation of monthly Salaries of all nurses is \$275.

$\sigma = 275$

Find 98% Conf. interval for mean monthly Salary of all nurses.

C-level: .98

σ Known $\Rightarrow$ Z Interval

$6099 < \mu < 6301$

$\bar{x} = \frac{+}{2} = 6200$

$E = \frac{-}{2} = 101$

I randomly selected 10 exams.

Here are the scores

82	75	90
100	70	95
68	78	87
	55	

1) $\bar{x} = 80$

2) $s \approx 14$

3) $n = 10$

} Round to a whole #

4) Find 99% Conf. interval for the mean of all exams.

$66 < \mu < 94$

σ unknown $\Rightarrow$ TInterval

inpt: STATS