Comparing Two Population Standard Deviations

$\sigma_1 \ & \ \sigma_2$
Why do we need to compare $\sigma_1$ and $\sigma_2$?

Whenever we gather sample data from two independent samples, we must know certain information about the standard deviations of the two populations.

- **Case I:** Two population standard deviations are both known,
- **Case II:** Two population standard deviations are both unknown,

  - They are assumed to be equal.
  - They are assumed to be not equal.
When do we need to worry about $\sigma_1$ and $\sigma_2$?

Here are two different occasions that we need to know these details.

- When we need to perform hypotheses testing about two population means.

- When we need to construct confidence interval for estimating the difference between two population means.
How do we compare $\sigma_1$ and $\sigma_2$?

We perform the F-Test for testing claims made about two population standard deviations.

Since we are interested to know whether two population standard deviations are equal or not, we only consider the Two-Tail Test process.

$$H_0 : \sigma_1 = \sigma_2$$

$$H_1 : \sigma_1 \neq \sigma_2$$
**Tips & Notations:**

- Choose Sample 1 & Sample 2 such that $s_1 > s_2$.
- Set up your Two-Column chart:

<table>
<thead>
<tr>
<th>Sample 1 (Numerator)</th>
<th>Sample 2 (Denominator)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1 =$</td>
<td>$s_2 =$</td>
</tr>
<tr>
<td>$n_1 =$</td>
<td>$n_2 =$</td>
</tr>
<tr>
<td>$Ndf =$</td>
<td>$Ddf =$</td>
</tr>
</tbody>
</table>
F-Test & TI:

- For Critical Values:
  \[\text{PRGM} \rightarrow \text{FVAL} \rightarrow \text{ENTER (Twice)}\]

- For Computed Test Statistic & P-Value:
  \[\text{STAT} \rightarrow \text{TESTS} \rightarrow 2\text{-SampFTest}\]

- Conclusion:
  - When \(H_0\) is valid \(\Rightarrow\) Assume \(\sigma_1 = \sigma_2\)
  - When \(H_1\) is valid \(\Rightarrow\) Assume \(\sigma_1 \neq \sigma_2\)
Example:
The standard deviation of ages of 15 randomly selected female students was 4.5 while the standard deviation of ages of 12 randomly selected male students was 7.2.

Use $\alpha = 0.02$ to test the claim that the standard deviation of ages of all female students is different from the standard deviation of ages of all male students.

Solution:
First, we select the sample of male students as our sample 1 since it has a larger standard deviation in compare with the sample of female students.

Now we are ready to make our chart.
Solution Continued:

<table>
<thead>
<tr>
<th>Sample 1: Males (Numerator)</th>
<th>Sample 2: Females (Denominator)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1 = 7.2$</td>
<td>$s_2 = 4.5$</td>
</tr>
<tr>
<td>$n_1 = 12$</td>
<td>$n_2 = 15$</td>
</tr>
<tr>
<td>$Ndf = 11$</td>
<td>$Ddf = 14$</td>
</tr>
</tbody>
</table>

Now we set up the null and alternative hypotheses.

$H_0 : \sigma_1 = \sigma_2$

$H_1 : \sigma_1 \neq \sigma_2$, TTT, Claim
Solution Continued:

Now we can use the program FVAL to find the critical values.

For the C.T.S. and P-Value, we can use 2-SampFTest.

- \( C.T.S. = 2.56 \implies C.T.S. \text{ is in } NCR \)
- \( P - Value = 0.101 \implies P - Value > \alpha \)
- \( H_0 \text{ is valid & } H_1 \text{ is invalid. } \implies \text{Reject the claim.} \)