Random variable

1) **Definition:** A variable whose value is the outcome of a random experiment

2) **Types:**

- **Discrete:** Assumes only countable values
- **Continuous:** Assumes any values

Probability Distribution of a Discrete Random Variable

- $0 \le P(x) \le 1$ for each x
- $\sum P(x) = 1$
- **Mean:** $\mu = \sum xP(x)$
- **Variance:** $\sigma^2 = \sum x^2 P(x) - \mu^2$
- **Standard Deviation:** $\sigma = \sqrt{\sigma^2} = \sqrt{\sum x^2 P(x) - \mu^2}$
- **Bar Graph:** $x-$values marked on the horizontal axis, $P(x)$ represents the height of the corresponding bar for each $x-$values on the vertical axis.

The Binomial Probability Distribution

- **Terms & Conditions:**

  1. n identical trials with only two possible outcomes for each trial
  2. Probability of the two outcomes remain constant.
  3. The trials are independent of each other.

- **Binomial Formula:** $P(x) = {}_nC_x p^x q^{n-x}$ where

  1. $n =$ total number of trials
  2. $p =$ probability of success, $0 \le p \le 1$
  3. $q = 1 - p =$ probability of failure, $0 \le q \le 1$
  4. $x =$ number of successes in n trials
  5. $n - x =$ number of failures in n trials
  6. ${}_nC_x = \dfrac{n!}{(n-x)!\,x!}$

- **Mean:** $\mu = np$

- **Variance:** $\sigma^2 = npq$

- **Standard Deviation:** $\sigma = \sqrt{\sigma^2} = \sqrt{npq}$

The Multinomial Distribution:

- **Terms & Conditions:**

1. n identical trials with more than two possible outcomes for each trial
2. Probability of each outcome remain constant.
3. The trials are independent and mutually exclusive of each other.

- **Multinomial Formula:**

$$P(x_1, x_2, \ldots, x_k) = \frac{n!}{x_1! \bullet x_2! \bullet \cdots \bullet x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$$

where

1. $n =$ total number of trials, $k =$ total number of events.

2. $P(E_1) = p_1, P(E_2) = p_2, P(E_3) = p_3$, and so on.

3. $0 \le p_k \le 1, \sum p_k = 1$

4. x_1 outcomes from event E_1, x_2 outcomes from event E_2, x_3 outcomes from event E_3, and so on from all k events.

The Hypergeometric Distribution:

- **Terms & Conditions:**

    1. Sampling from small population **<u>without</u>** replacement.
    2. The trials are not independent from each other.
    3. The outcomes belong to one of the two types.

- **Binomial Formula:** $P(x) = \dfrac{{}_{r}C_{x} \bullet {}_{N-r}C_{n-x}}{{}_{N}C_{n}}$ where

    1. $N =$ Size of the population

    2. $n =$ Size of the sample(Number of trials)

    3. $r =$ Number of successes in the population

    4. $x =$ Number of successes in n trials.

    5. $N - n =$ Number of failures in the population

    6. $n - x =$ Number of failures in n trials

    7. ${}_{n}C_{x} = \dfrac{n!}{(n-x)!x!}$

The Poisson Distribution:

- **Terms & Conditions:**

  1. Applies to occurrences over a specified interval.
  2. The occurrences are random, independent, and uniformly distributed over the specified interval.

- **The Poisson Distribution Formulas:**

  1. $P(x) = \dfrac{\mu^x e^{-\mu}}{x!}$ where
     - $\mu =$ mean
     - x is the number of occurrences of an event over the interval being used.
     - $e \approx 2.71828$
  2. The standard deviation is $\sigma = \sqrt{\mu}$
  3. Examples of intervals: time, distance, area, volume, or some similar units.
  4. You may use the Poisson Distribution as an approximation to the Binomial distribution when n is large($n \geq 100$) **<u>and</u>** p is small($np \leq 10$).