Chapter 8
Hypothesis Testing

Slide 1

Overview

Slide 2

Definition

- In statistics, a **hypothesis** is a claim or statement about a property of a population.
- A **hypothesis test** (or **test of significance**) is a standard procedure for testing a claim about a property of a population.

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Components of a Formal Hypothesis Test

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Null Hypothesis: H_0

- The ***null hypothesis*** includes the assumed value of the population parameter.
- It must be a statement of equality.
- Test the Null Hypothesis **directly**
- **Reject** H_0 or **fail to reject** H_0

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Components of a Formal Hypothesis Test

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Alternative Hypothesis: H_1

- The **alternative hypothesis** (denoted by H_1 or H_a) is the statement that the parameter has a value that somehow differs from the null hypothesis.
- $\neq$, <, >

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Note about Forming Your Own Claims (Hypotheses)

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If you are conducting a study and want to use a hypothesis test to **support** your claim, the claim must be worded so that it becomes the **alternative hypothesis.**

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Note about Identifying H_0 and H_1

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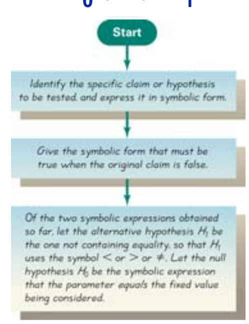

Figure 7-2

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Slide 7

Example: **Identify the Null and Altenative Hypothesis. Refer to Figure 7-2 and use the given claims to express the corresponding null and alternative hypotheses in symbolic form.**

1) The proportion of drivers who admit to running red lights is greater than 0.5.

In Step 1 of Figure 7-2, we express the given claim as $p > 0.5$. In Step 2, we see that if $p > 0.5$ is false, then $p \le 0.5$ must be true. In Step 3, we see that the expression $p > 0.5$ does not contain equality, so we let the alternative hypothesis H_1 be $p > 0.5$, and we let H_0 be $p = 0.5$.

Right-tailed Test

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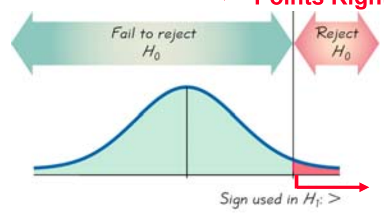
$$H_0 : p = .5$$

$$H_1 : p > 0.5$$

Points Right



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Example: **Identify the Null and Altenative Hypothesis. Refer to Figure 7-2 and use the given claims to express the corresponding null and alternative hypotheses in symbolic form.**

2) The mean height of professional basketball players is at least 6 ft.

In Step 1 of Figure 7-2, we express "a mean of at least 6 ft" in symbols $\mu \ge 6$. In Step 2, we see that if $\mu \ge 6$ is false, then $\mu < 6$ must be true. In Step 3, we see that the expression $\mu < 6$ does not contain equality, so we let the alternative hypothesis H_1 be $\mu < 6$, and we let H_0 be $\mu \ge 6$.

Left-tailed Test

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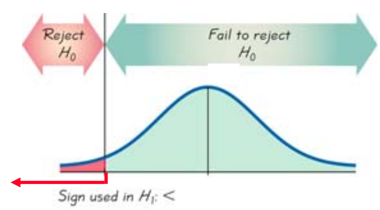
$$H_0 : \mu = 6$$

$$H_1 : \mu < 6$$

Points Left



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Example: **Identify the Null and Altenative Hypothesis. Refer to Figure 7-2 and use the given claims to express the corresponding null and alternative hypotheses in symbolic form.**

3) The standard deviation of IQ scores of actors is equal to 15.

In Step 1 of Figure 7-2, we express the given claim as $\sigma = 15$. In Step 2, we see that if $\sigma = 15$ is false, then $\sigma \ne 15$ must be true. In Step 3, we let the alternative hypothesis H_1 be $\sigma \ne 15$, and we let H_0 be $\sigma = 15$.

Two-tailed Test

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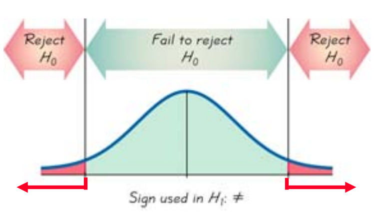
$$H_0 : \sigma = 15$$

$$H_1 : \sigma \ne 15$$

α is divided equally between the two tails of the critical region

less than or greater than



Test Statistic

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The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

Test statistic for proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Test Statistic

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The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

Test statistic for mean

$$z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

When σ is known

Test Statistic

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The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

Test statistic for mean

When σ is unknown

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Test Statistic

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The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

Test statistic for standard deviation

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Critical Region

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The critical region (or rejection region) is the set of all values of the test statistic that cause us to reject the null hypothesis. For example, see the red-shaded region in Figure 7-3.

Significance Level

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The significance level (denoted by α) is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true. This is the same α introduced in Section 6-2. Common choices for α are 0.05, 0.01, and 0.10.

Critical Value

A **critical value** is any value that separates the critical region (where we reject the null hypothesis) from the non-critical region which is determined by the value of the significance level α.

P-Value

The ***P*-value** (or ***p*-value** or **probability value**) is the probability of getting a value of the test statistic that is *at least as extreme* as the one representing the sample data, assuming that the null hypothesis is true.

How to find P-values

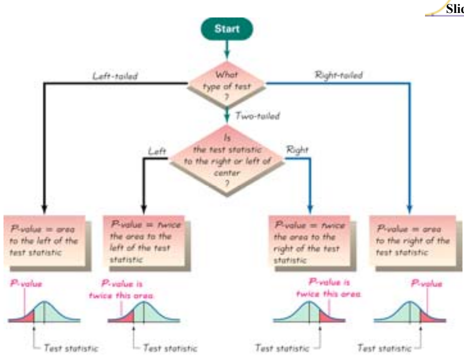

Figure 7-6

Conclusions in Hypothesis Testing

- We always test the null hypothesis.

1. Reject the H_0
2. Fail to reject the H_0

Decision Criterion

Traditional method:

Reject H_0 if the test statistic falls within the critical region.

Fail to *reject H_0* if the test statistic does not fall within the critical region.

Decision Criterion

P-value method:

Reject H_0 if *P*-value $\leq \alpha$.

Fail to *reject H_0* if *P*-value $> \alpha$.

Decision Criterion

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Another option:

Instead of using a significance level such as 0.05, simply identify the *P*-value and leave the decision to the reader.

Wording of Final Conclusion

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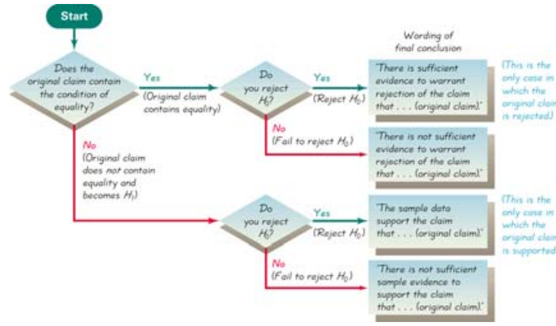

Figure 7-7

Accept versus Fail to Reject

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- **Some texts use "accept the null hypothesis."**
- **We are not proving the null hypothesis.**
- **The sample evidence is not strong enough to warrant rejection (such as not enough evidence to convict a suspect).**

Type I & II Error

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- **A Type I error is the mistake of rejecting the null hypothesis when it is true.**
- **The symbol α (alpha) is used to represent the probability of a type I error.**
- **A Type II error is the mistake of failing to reject the null hypothesis when it is false.**
- **The symbol β(beta) is used to represent the probability of a type II error.**

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Example: **Assume that we a conducting a hypothesis test of the claim $p > 0.5$. Here are the null and alternative hypotheses: H_0: $p = 0.5$, and H_1: $p > 0.5$.**

a) A type I error is the mistake of rejecting a true null hypothesis, so this is a type I error: Conclude that there is sufficient evidence to support $p > 0.5$, when in reality $p = 0.5$.

b) A type II error is the mistake of failing to reject the null hypothesis when it is false, so this is a type II error: Fail to reject $p = 0.5$ (and therefore fail to support $p > 0.5$) when in reality $p > 0.5$.

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Table 7-1 Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true.	The null hypothesis is false.
Decision	We decide to reject the null hypothesis.	Type I error (rejecting a true null hypothesis) α	Correct decision
	We fail to reject the null hypothesis.	Correct decision	Type II error (failing to reject a false null hypothesis) β

Controlling Type I and Type II Errors

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- For any fixed α, an increase in the sample size n will cause a decrease in β.
- For any fixed sample size n , a decrease in α will cause an increase in β. Conversely, an increase in α will cause a decrease in β .
- To decrease both α and β, increase the sample size.

Assumptions for Testing Claims About a Population Proportion *p*

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1) The sample observations are a simple random sample.
2) The conditions for a binomial experiment are satisfied (Section 4-3)

Assumptions for Testing Claims About a Population Proportion *p*

Slide 33

1) The sample observations are a simple random sample.
2) The conditions for a binomial experiment are satisfied (Section 4-3)
3) The condition $np \geq 5$ and $nq \geq 5$ are satisfied, so the binomial distribution of sample proportions can be approximated by a normal distribution with

$\mu = np$ and $\sigma = \sqrt{npq}$.

Notation

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n = number of trials

$\hat{p} = \frac{x}{n}$ (*sample* proportion)

p = population proportion (used in the null hypothesis)

$q = 1 - p$

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$\hat{p}$ sometimes is given directly

"10% of the observed sports cars are red" is expressed as

$$\hat{p} = 0.10$$

$\hat{p}$ sometimes must be calculated

"96 surveyed households have cable TV and 54 do not" is calculated using

$$\hat{p} = \frac{x}{n} = \frac{96}{(96+54)} = 0.64$$

(determining the sample proportion of households with cable TV)

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Example: In the Chapter Problem, we noted that an article distributed by the Associated Press included these results from a nationwide survey: Of 880 randomly selected drivers, 56% admitted that they run red lights. The claim is that the majority of all Americans run red lights. That is, $p > 0.5$.

The sample data are $n = 880$, and $\hat{p} = 0.56$.

$np = (880)(0.5) = 440 \geq 5$
$nq = (880)(0.5) = 440 \geq 5$
and

$$x = n\hat{p} = 880(.56) = 492.8 \approx 493$$

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Example: In the Chapter Problem, we noted that an article distributed by the Associated Press included these results from a nationwide survey: Of 880 randomly selected drivers, 56% admitted that they run red lights. The claim is that the majority of all Americans run red lights. That is, $p > 0.5$. The sample data are $n = 880$, and $\hat{p} = 0.56$. We will use the Traditional Method.

H_0: $p = 0.5$
H_1: $p > 0.5$
$\alpha = 0.05$

$$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.56-0.5}{\sqrt{\frac{(0.5)(0.5)}{880}}} = 3.56$$

This is a right-tailed test, so the critical region is an area of 0.05. We find that $z = 1.645$ is the critical value of the critical region. We reject the null hypothesis. There is sufficient evidence to support the claim.

Slide 38

Example: In the Chapter Problem, we noted that an article distributed by the Associated Press included these results from a nationwide survey: Of 880 randomly selected drivers, 56% admitted that they run red lights. The claim is that the majority of all Americans run red lights. That is, $p > 0.5$. The sample data are $n = 880$, and $\hat{p} = 0.56$. We will use the *P*-value Method.

H_0: $p = 0.5$
H_1: $p > 0.5$
$\alpha = 0.05$

$$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.56-0.5}{\sqrt{\frac{(0.5)(0.5)}{880}}} = 3.56$$

Referring to Table A-2, we see that for values of $z = 3.50$ and higher, we use 0.9999 for the cumulative area to the left of the test statistic. The P-value is 1 – 0.9999 = 0.0001.

Slide 39

Example: In the Chapter Problem, we noted that an article distributed by the Associated Press included these results from a nationwide survey: Of 880 randomly selected drivers, 56% admitted that they run red lights. The claim is that the majority of all Americans run red lights. That is, $p > 0.5$. The sample data are $n = 880$, and $\hat{p} = 0.56$. We will use the *P*-value Method.

H_0: $p = 0.5$
H_1: $p > 0.5$
$\alpha = 0.05$

$$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.56-0.5}{\sqrt{\frac{(0.5)(0.5)}{880}}} = 3.56$$

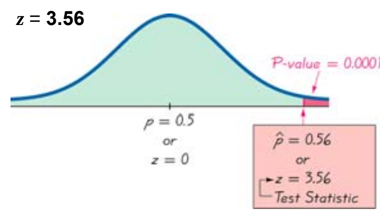
Since the *P*-value of 0.0001 is less than the significance level of $\alpha = 0.05$, we reject the null hypothesis. There is sufficient evidence to support the claim.

Slide 40

Example: In the Chapter Problem, we noted that an article distributed by the Associated Press included these results from a nationwide survey: Of 880 randomly selected drivers, 56% admitted that they run red lights. The claim is that the majority of all Americans run red lights. That is, $p > 0.5$. The sample data are $n = 880$, and $\hat{p} = 0.56$. We will use the *P*-value Method.

H_0: $p = 0.5$
H_1: $p > 0.5$
$\alpha = 0.05$

$z = 3.56$



Using TI Calculator:

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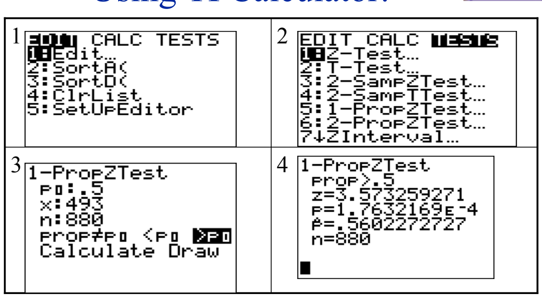

Compare these results with earlier answers.

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Example: In the Chapter Problem, we noted that an article distributed by the Associated Press included these results from a nationwide survey: Of 880 randomly selected drivers, 56% admitted that they run red lights. The claim is that the majority of all Americans run red lights. That is, $p > 0.5$. The sample data are $n = 880$, and $\hat{p} = 0.56$. We will use the confidence interval method.

For a one-tailed hypothesis test with significance level α, we will construct a confidence interval with a confidence level of $1 - 2\alpha$. Using the methods from Section 6-2, we construct a 90% confidence interval.

We obtain $0.533 < p < 0.588$. We are 90% confident that the true value of p is contained within the limits of 0.533 and 0.588. Thus we support the claim that $p > 0.5$.

CAUTION

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- When the calculation of $\hat{p}$ results in a decimal with many places, store the number on your calculator and use all the decimals when evaluating the z test statistic.
- Large errors can result from rounding $\hat{p}$ too much.

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Example: When Gregory Mendel conducted his famous hybridization experiments with peas, one such experiment resulted in offspring consisting of 428 peas with green pods and 152 peas with yellow pods. According to Mendel's theory, 1/4 of the offspring peas should have yellow pods. Use a 0.05 significance level with the *P*-value method to test the claim that the proportion of peas with yellow pods is equal to 1/4.

We note that n = 428 + 152 = 580, so $\hat{p}$ = 0.262, and p = 0.25.

Slide 45

Example: When Gregory Mendel conducted his famous hybridization experiments with peas, one such experiment resulted in offspring consisting of 428 peas with green pods and 152 peas with yellow pods. According to Mendel's theory, 1/4 of the offspring peas should have yellow pods. Use a 0.05 significance level with the *P*-value method to test the claim that the proportion of peas with yellow pods is equal to 1/4.

H_0: $p = 0.25$
H_1: $p \neq 0.25$
$n = 580$
$\alpha = 0.05$
$\hat{p} = 0.262$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.262 - 0.25}{\sqrt{\frac{(0.25)(0.75)}{580}}} = 0.67$$

Since this is a two-tailed test, the *P*-value is twice the area to the right of the test statistic. Using Table A-2, *z* = 0.67 is 1 – 0.7486 = 0.2514.

Slide 46

Example: When Gregory Mendel conducted his famous hybridization experiments with peas, one such experiment resulted in offspring consisting of 428 peas with green pods and 152 peas with yellow pods. According to Mendel's theory, 1/4 of the offspring peas should have yellow pods. Use a 0.05 significance level with the *P*-value method to test the claim that the proportion of peas with yellow pods is equal to 1/4.

H_0: $p = 0.25$
H_1: $p \neq 0.25$
$n = 580$
$\alpha = 0.05$
$\hat{p} = 0.262$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.262 - 0.25}{\sqrt{\frac{(0.25)(0.75)}{580}}} = 0.67$$

The *P*-value is 2(0.2514) = 0.5028. We fail to reject the null hypothesis. There is not sufficient evidence to warrant rejection of the claim that 1/4 of the peas have yellow pods.

Using TI Calculator:

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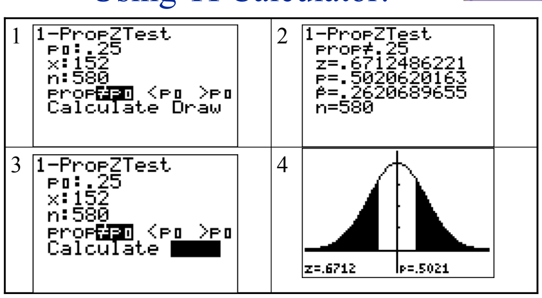

Compare these results with earlier answers.

Test Statistic for Testing a Claim about a Proportion

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$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - np}{\sqrt{npq}} = \frac{\frac{x}{n} - \frac{np}{n}}{\frac{\sqrt{npq}}{n}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Assumptions for Testing Claims About Population Means

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1) The sample is a simple random sample.

2) The value of the population standard deviation σ is known.

3) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

Example: Data Set 4 in Appendix B lists a sample of 106 body temperatures having a mean of 98.20°F. Assume that the sample is a simple random sample and that the population standard deviation σ is known to be 0.62°F. Use a 0.05 significance level to test the common belief that the mean body temperature of healthy adults is equal to 98.6°F. Use the *P*-value method.

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H_0: $\mu = 98.6$
H_1: $\mu \neq 98.6$
$\alpha = 0.05$
$\bar{x} = 98.2$
$\sigma = 0.62$

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}} = \frac{98.2 - 98.6}{\frac{0.62}{\sqrt{106}}} = -6.64$$

This is a two-tailed test and the test statistic is to the left of the center, so the *P*-value is twice the area to the left of $z = -6.64$. We refer to Table A-2 to find the area to the left of $z = -6.64$ is 0.0001, so the *P*-value is 2(0.0001) = 0.0002.

Example: Data Set 4 in Appendix B lists a sample of 106 body temperatures having a mean of 98.20°F. Assume that the sample is a simple random sample and that the population standard deviation σ is known to be 0.62°F. Use a 0.05 significance level to test the common belief that the mean body temperature of healthy adults is equal to 98.6°F. Use the *P*-value method.

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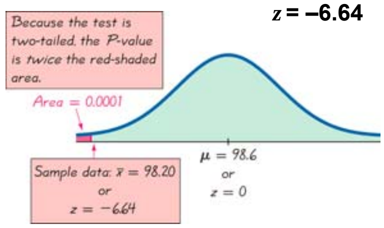
H_0: $\mu = 98.6$
H_1: $\mu \neq 98.6$
$\alpha = 0.05$
$\bar{x} = 98.2$
$\sigma = 0.62$

$z = -6.64$



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Example: Data Set 4 in Appendix B lists a sample of 106 body temperatures having a mean of 98.20°F. Assume that the sample is a simple random sample and that the population standard deviation σ is known to be 0.62°F. Use a 0.05 significance level to test the common belief that the mean body temperature of healthy adults is equal to 98.6°F. Use the *P*-value method.

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H_0: $\mu = 98.6$
H_1: $\mu \neq 98.6$
$\alpha = 0.05$
$\bar{x} = 98.2$
$\sigma = 0.62$

$z = -6.64$

Because the *P*-value of 0.0002 is less than the significance level of $\alpha = 0.05$, we reject the null hypothesis.
There is sufficient evidence to conclude that the mean body temperature of healthy adults differs from 98.6°F.

Example: Data Set 4 in Appendix B lists a sample of 106 body temperatures having a mean of 98.20°F. Assume that the sample is a simple random sample and that the population standard deviation σ is known to be 0.62°F. Use a 0.05 significance level to test the common belief that the mean body temperature of healthy adults is equal to 98.6°F. Use the Traditional method.

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$z = -6.64$

H_0: $\mu = 98.6$
H_1: $\mu \neq 98.6$
$\alpha = 0.05$
$\bar{x} = 98.2$
$\sigma = 0.62$

We now find the critical values to be $z = -1.96$ and $z = 1.96$. We would reject the null hypothesis, since the test statistic of $z = -6.64$ would fall in the critical region.

There is sufficient evidence to conclude that the mean body temperature of healthy adults differs from 98.6°F.

Using TI Calculator:

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1 EDIT CALC TESTS 1:Z-Test… 2:T-Test… 3:2-SampZTest… 4:2-SampTTest… 5:1-PropZTest… 6:2-PropZTest… 7↓ZInterval…	2 Z-Test Inpt:Data Stats μ0:98.6 σ:.62 x̄:98.2 n:106 μ:≠μ0 <μ0 >μ0 Calculate Draw
3 Z-Test μ≠98.6 z=-6.642342026 p=3.1039E-11 x̄=98.2 n=106	4 Compare the sample mean with the 95% confidence interval, is this consistent with our conclusion?

Compare these results with earlier answers.

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Assumptions for Testing Claims About a Population Mean (with σ Not Known)

1) The sample is a simple random sample.

2) The value of the population standard deviation σ is *not* known.

3) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$.

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Important Properties of the Student *t* Distribution

1. The Student *t* distribution is different for different sample sizes (see Figure 6-5 in Section 6-4).
2. The Student *t* distribution has the same general bell shape as the normal distribution; its wider shape reflects the greater variability that is expected when s is used to test σ.
3. The Student *t* distribution has a mean of $t = 0$ (just as the standard normal distribution has a mean of $z = 0$).
4. The standard deviation of the Student *t* distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has a $\sigma = 1$).
5. As the sample size *n* gets larger, the Student *t* distribution get closer to the normal distribution.

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Choosing between the Normal and Student *t* Distributions when Testing a Claim about a Population Mean μ

Use the Student *t* distribution when σ is not known and either or both of these conditions is satisfied:
Population is normally distributed

OR

$n > 30$.

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Example: A premed student in a statistics class is required to do a class project. She plans to collect her own sample data to test the claim that the mean body temperature is less than 98.6°F. After carefully planning a procedure for obtaining a simple random sample of 12 healthy adults, she measures their body temperatures and obtains the results on page 409. Use a 0.05 significance level to test the claim these body temperatures come from a population with a mean that is less than 98.6°F.
Use the Traditional method.

There are no outliers, and based on a histogram and normal quantile plot, we can assume that the data are from a population with a normal distribution.
We use the sample data to find $n = 12$, $\bar{x} = 98.39$, and $s = 0.535$.

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Example: A premed student in a statistics class is required to do a class project. She plans to collect her own sample data to test the claim that the mean body temperature is less than 98.6°F. After carefully planning a procedure for obtaining a simple random sample of 12 healthy adults, she measures their body temperatures and obtains the results on page 409. Use a 0.05 significance level to test the claim these body temperatures come from a population with a mean that is less than 98.6°F.
Use the Traditional method.

H_0: $\mu = 98.6$
H_1: $\mu < 98.6$
$\alpha = 0.05$
$\bar{x} = 98.39$
$s = 0.535$
$n = 12$

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}} = \frac{98.39 - 98.6}{\frac{0.535}{\sqrt{12}}} = -1.360$$

Slide 60

Example: A premed student in a statistics class is required to do a class project. She plans to collect her own sample data to test the claim that the mean body temperature is less than 98.6°F. After carefully planning a procedure for obtaining a simple random sample of 12 healthy adults, she measures their body temperatures and obtains the results on page 409. Use a 0.05 significance level to test the claim these body temperatures come from a population with a mean that is less than 98.6°F.
Use the Traditional method.

H_0: $\mu = 98.6$
H_1: $\mu < 98.6$
$\alpha = 0.05$
$\bar{x} = 98.39$
$s = 0.535$
$n = 12$

$t = -1.360$

Because the test statistic of $t = -1.360$ does not fall in the critical region, we fail to reject H_0. There is not sufficient evidence to to support the claim that the sample comes from a population with a mean less than 98.6°F.

Example:

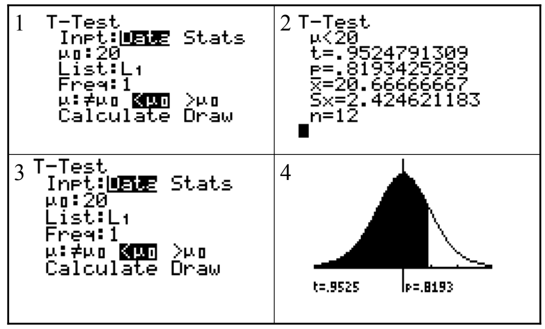
12 students quiz scores are given below:

25, 20, 18, 20, 19, 17, 24, 23, 20, 21, 19, 22

Assume scores are normally distributed, test the claim that the mean quiz score of all student is less than 20, use $\alpha = 0.01$.

Enter data into L_1, then since σ is unknown, we use T-test.

Using TI Calculator:



Assumptions for Testing Claims About σ or σ^2

1. The sample is a simple random sample.
2) The population has values that are normally distributed (a strict requirement).

Chi-Square Distribution

Test Statistic

$$\chi^2 = \frac{(n-1)\,s^2}{\sigma^2}$$

n = sample size

s^2 = sample variance

σ^2 = population variance (given in null hypothesis)

P-values and Critical Values for Chi-Square Distribution

- Use Table A-4.
- The degrees of freedom = $n - 1$.

Properties of Chi-Square Distribution

- All values of χ^2 are nonnegative, and the distribution is not symmetric (see Figure 7-12).
- There is a different distribution for each number of degrees of freedom (see Figure 7-13).
- The critical values are found in Table A-4 using $n - 1$ degrees of freedom.

Properties of Chi-Square Distribution

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Properties of the Chi-Square Distribution

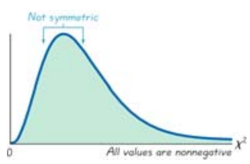
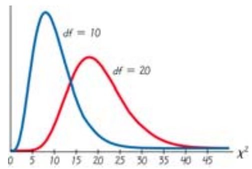


Figure 7-12

Chi-Square Distribution for 10 and 20 Degrees of Freedom



There is a different distribution for each number of degrees of freedom.

Figure 7-13

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Example: **For a simple random sample of adults, IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. A simple random sample of 13 statistics professors yields a standard deviation of *s* = 7.2. Assume that IQ scores of statistics professors are normally distributed and use a 0.05 significance level to test the claim that σ = 15.**

H_0: σ = 15
H_1: σ ≠ 15
α = 0.05
n = 13
s = 7.2

$$\chi^2 = \frac{(n-1)\,s^2}{\sigma^2} = \frac{(13-1)(7.2)^2}{15^2} = 2.765$$

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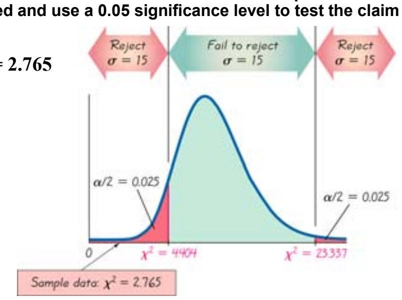
Example: **For a simple random sample of adults, IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. A simple random sample of 13 statistics professors yields a standard deviation of *s* = 7.2. Assume that IQ scores of statistics professors are normally distributed and use a 0.05 significance level to test the claim that σ = 15.**

H_0: σ = 15
H_1: σ ≠ 15
α = 0.05
n = 13
s = 7.2

$\chi^2 = 2.765$



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Example: **For a simple random sample of adults, IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. A simple random sample of 13 statistics professors yields a standard deviation of *s* = 7.2. Assume that IQ scores of statistics professors are normally distributed and use a 0.05 significance level to test the claim that σ = 15.**

H_0: σ = 15
H_1: σ ≠ 15
α = 0.05
n = 13
s = 7.2

$\chi^2 = 2.765$

The critical values of 4.404 and 23.337 are found in Table A-4, in the 12th row (degrees of freedom = n – 1) in the column corresponding to 0.975 and 0.025.

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Example: **For a simple random sample of adults, IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. A simple random sample of 13 statistics professors yields a standard deviation of *s* = 7.2. Assume that IQ scores of statistics professors are normally distributed and use a 0.05 significance level to test the claim that σ = 15.**

H_0: σ = 15
H_1: σ ≠ 15
α = 0.05
n = 13
s = 7.2

$\chi^2 = 2.765$

Because the test statistic is in the critical region, we reject the null hypothesis. There is sufficient evidence to warrant rejection of the claim that the standard deviation is equal to 15.