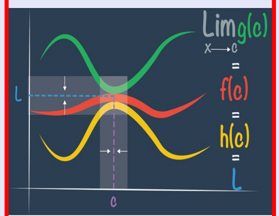
Calculus I

Lecture 9



Feb 19-8:47 AM

Class QZ 5 Box Your Final Ans

1) $\lim_{x\to 2}\frac{x^3-8}{x-2} = \frac{0}{0}$ I.F. $\lim_{x\to 2}\frac{x^3-8}{x-2} = \lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$

$= \lim_{x\to 2}(x^2+2x+4) = \boxed{12}$ ✓

2) $\lim_{x\to\infty}\frac{2x+5}{x-1} = \frac{\infty}{\infty}$ I.F. $\lim_{x\to\infty}\frac{2x+5}{x-1} = \lim_{x\to\infty}\frac{2+\frac{5}{x}\to 0}{1-\frac{1}{x}\to 0} = \frac{2}{1} = \boxed{2}$ ✓

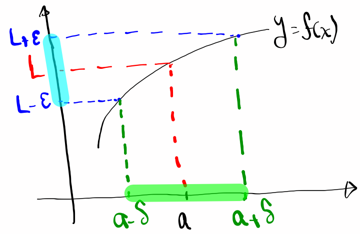
3) $\lim_{x\to 16}\frac{\sqrt{x}-4}{x-16} = \frac{0}{0}$ I.F. $\lim_{x\to 16}\frac{\sqrt{x}-4}{x-16} = \lim_{x\to 16}\frac{(\sqrt{x}-4)(\sqrt{x}+4)}{(x-16)(\sqrt{x}+4)}$

$= \lim_{x\to 16}\frac{\cancel{x-16}}{\cancel{(x-16)}(\sqrt{x}+4)} = \lim_{x\to 16}\frac{1}{\sqrt{x}+4}$

$= \frac{1}{4+4}$

$= \boxed{\frac{1}{8}}$ ✓

Feb 15-9:44 AM



Any value for x in $(a-\delta, a+\delta)$

the function value $f(x)$ is in $(L-\varepsilon, L+\varepsilon)$

Precise def. of limits: $\boxed{\varepsilon>0\ ,\ \delta>0}$

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

For every $\varepsilon>0$, there is a $\delta>0$ such that
$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

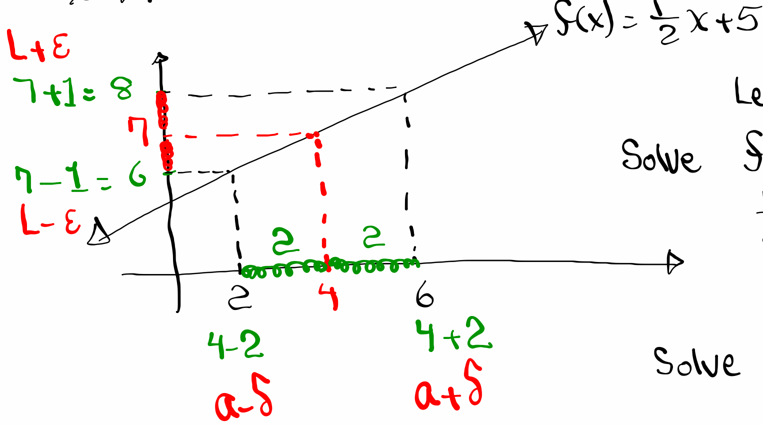
Our goal is to find a δ for every ε.

Find a relationship between ε & δ.

Feb 20-8:54 AM

$$f(x) = \frac{1}{2}x + 5$$

$$\lim_{x\to 4} f(x) = \lim_{x\to 4}\left(\frac{1}{2}x+5\right) = \frac{1}{2}(4) + 5 = 2+5 = 7$$



Let's pick $\varepsilon=1$

Solve $f(x)=8$

$\frac{1}{2}x+5=8$

$\frac{1}{2}x=3 \rightarrow \boxed{x=6}$

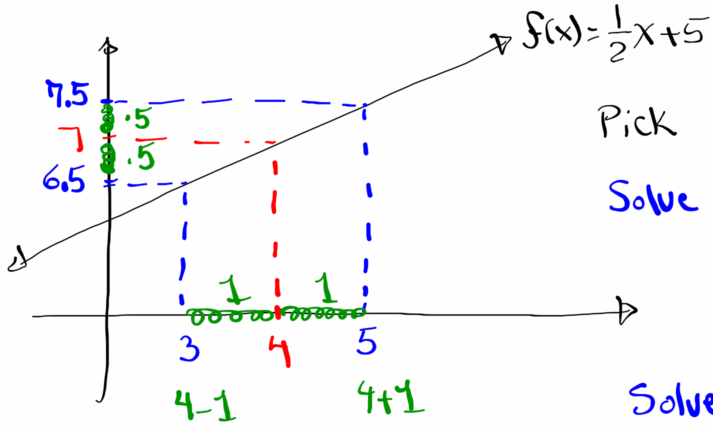
Solve $f(x)=6$

$\frac{1}{2}x+5=6$

$\frac{1}{2}x=1 \rightarrow x=2$

For $\varepsilon=1 \rightarrow$ choose $\delta=2$

Feb 20-9:02 AM



Pick $\varepsilon=.5$

Solve $f(x)=7.5$

$\frac{1}{2}x+5=7.5$

$\boxed{x=5}$

Solve $f(x)=6.5$

$\frac{1}{2}x+5=6.5$

$\boxed{x=3}$

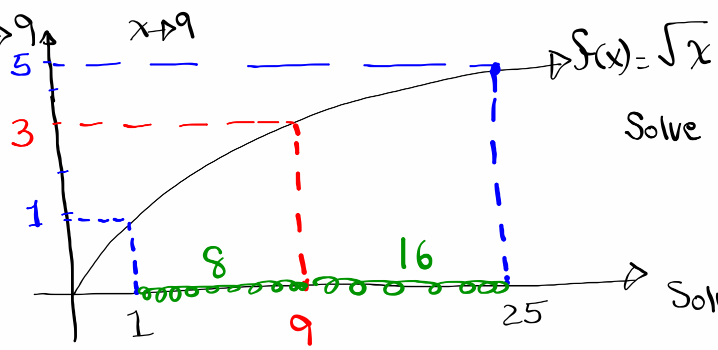
For $\varepsilon=.5$, we choose $\delta=1$

Feb 20-9:08 AM

$f(x)=\sqrt{x}$

$$\lim_{x\to 9} f(x)=\lim_{x\to 9}\sqrt{x}=\sqrt{9}=3$$

$\varepsilon=2$



Solve $f(x)=5$

$\sqrt{x}=5$

$x=25$

Solve $f(x)=1$

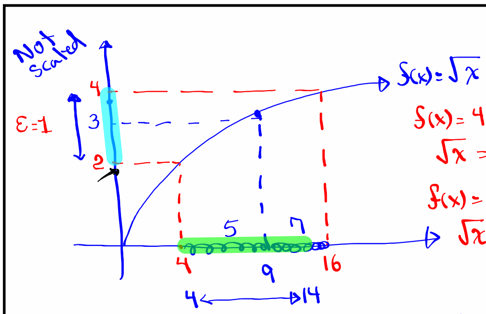
$\sqrt{x}=1$

$x=1$

Pick $\delta=\min\{8,16\}=8$

For $\varepsilon=2$, we choose $\delta=8$.

Feb 20-9:12 AM



$f(x)=4$

$\sqrt{x}=4 \to x=16$

$f(x)=2$

$\sqrt{x}=2 \to x=4$

For $\varepsilon=1$, we choose $\delta=\min\{5,7\}=\boxed{5}$

For $\varepsilon=1$, we choose $\delta=5$.

$a=9$

$(a-\delta\ ,\ a+\delta)$

$(9-5\ ,\ 9+5)$

$(4,\ 14)$

Suppose $x=13.5$

$\sqrt{13.5}\approx 3.7 \to (2,4)$

Suppose $x=4.1$

$\sqrt{4.1}\approx 2.^{+} \to (2,4)$

Suppose we take $\cancel{\delta=6}$.

$(a-\delta\ ,\ a+\delta)$

$(9-6\ ,\ 9+6)$

$(3\ ,\ 15)$

$\sqrt{14.5}\approx 3.8 \to (2,4)$

$\sqrt{3.1}\approx 1.8 \to (2,4)$

Not within the required interval

Feb 20-9:18 AM

Use $\boxed{\text{definition of limits}}$ to prove $\lim_{x\to 3}(4x-1)=11$

$f(x)=4x-1$

$a=3$

$L=11$

1) verify

$\lim_{x\to 3}(4x-1)=11$?

$\lim_{x\to 3}(4x-1)=4(3)-1=11$ ✓

$\boxed{\text{For every } \varepsilon>0\text{, there is a } \delta>0 \text{ such that } |f(x)-L|<\varepsilon \text{ whenever } |x-a|<\delta}$

$|4x-1-11|<\varepsilon$ whenever $|x-3|<\delta$

$|4x-12|<\varepsilon$ 〃 $|x-3|<\delta$

$|4(x-3)|<\varepsilon$ 〃 $|x-3|<\delta$

$|ab|=|a||b|$

$|4|\,|x-3|<\varepsilon$

$4|x-3|<\varepsilon \to |x-3|<\frac{\varepsilon}{4}$

Pick $\boxed{\delta=\frac{\varepsilon}{4}}$

Isolate $|x-3|$

If $\varepsilon=1 \to \delta=\frac{1}{4}$

If $\varepsilon=2 \to \delta=\frac{2}{4}=\frac{1}{2}$

If $\varepsilon=.5 \to \delta=\frac{1}{8}$

If $\varepsilon=.1 \to \delta=\frac{1}{40}$

Feb 20-9:29 AM

Use def. of limit to prove $\lim_{x \to 4} \left(\frac{1}{2}x+5\right)=7$

$f(x)=\frac{1}{2}x+5$

$a=4$

$L=7$

Verify $\lim_{x \to a} f(x)=L$

$\lim_{x \to 4}\left(\frac{1}{2}x+5\right)=\frac{1}{2}(4)+5=2+5=7 \checkmark$

$\boxed{\text{For every } \varepsilon>0 \text{, there is a } \delta>0 \text{ such that } |f(x)-L|<\varepsilon \text{ whenever } |x-a|<\delta}$

$\left|\frac{1}{2}x+5-7\right|<\varepsilon$ whenever $|x-4|<\delta$

$\left|\frac{1}{2}x-2\right|<\varepsilon$ 〃 $|x-4|<\delta$

$\left|\frac{1}{2}(x-4)\right|<\varepsilon$ 〃 $|x-4|<\delta$

$|ab|=|a||b|$

$\left|\frac{1}{2}\right||x-4|<\varepsilon$

$\frac{1}{2}|x-4|<\varepsilon$

Isolate

Multiply by 2

$|x-4|<2\varepsilon$

$\boxed{\text{Pick } \delta=2\varepsilon}$

If $\varepsilon=1 \to \delta=2$

If $\varepsilon=2 \to \delta=4$

If $\varepsilon=\frac{1}{2} \to \delta=1$

If $\varepsilon=\frac{1}{4} \to \delta=\frac{1}{2}$

Feb 20-9:39 AM

Prove $\lim_{x \to 3} x^2=9$

$f(x)=x^2$

$a=3$

$L=9$

Verify $\lim_{x \to 3} x^2=9$

$3^2=9 \checkmark$

For every $\varepsilon>0$, there is a $\delta>0$ such that

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^2-9|<\varepsilon$ 〃 $|x-3|<\delta$

$|(x+3)(x-3)|<\varepsilon$ 〃 $|x-3|<\delta$

$|ab|=|a||b|$

$|x+3||x-3|<\varepsilon$

Isolate

Suppose $|x+3|<C$

$C|x-3|<\varepsilon$

$|x-3|<\frac{\varepsilon}{C}$

$\delta=\frac{\varepsilon}{C}$

Finish it on Wednesday.

Google Squeeze theorem

Feb 20-9:49 AM