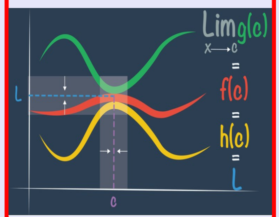
Calculus I

Lecture 8



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Limit Rules:

Suppose $\lim_{x \to a} f(x) = L_1$ and $\lim_{x \to a} g(x) = L_2$

1) $\lim_{x \to a} \left[f(x) + g(x) \right] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L_1 + L_2$

2) $\lim_{x \to a} \left[f(x) - g(x) \right] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L_1 - L_2$

3) $\lim_{x \to a} \left[f(x) \cdot g(x) \right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = L_1 \cdot L_2$

4) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \dfrac{L_1}{L_2}$ if $L_2 \neq 0$

5) $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L_1}$ if n is even, then $L_1 \geq 0$

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6) $\lim\limits_{x \to a} [f(x)]^n = \left[\lim\limits_{x \to a} f(x)\right]^n = L_1^n$

7) $\lim\limits_{x \to a} K f(x) = \lim\limits_{x \to a} K \cdot \lim\limits_{x \to a} f(x) = K \cdot L_1$

constant

ex: Evaluate $\lim\limits_{x \to 2} (x^2 - 5x + 3) =$

$\lim\limits_{x \to 2} x^2 - \lim\limits_{x \to 2} 5x + \lim\limits_{x \to 2} 3 =$

$\left[\lim\limits_{x \to 2} x\right]^2 - \lim\limits_{x \to 2} 5 \cdot \lim\limits_{x \to 2} x + \lim\limits_{x \to 2} 3 =$

$2^2 - 5 \cdot 2 + 3 = 4 - 10 + 3$

$= \boxed{-3}$

$$\boxed{\begin{array}{l} \lim\limits_{x \to a} x = a \\ \lim\limits_{x \to a} K = K \end{array}}$$

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$\lim\limits_{x \to -2} \dfrac{5x^3 + 4}{x - 3} = \dfrac{\lim\limits_{x \to -2} (5x^3 + 4)}{\lim\limits_{x \to -2} (x - 3)}$

$= \dfrac{\lim\limits_{x \to -2} 5x^3 + \cancel{\lim\limits_{x \to -2} 4} \to 4}{\cancel{\lim\limits_{x \to -2} x} \to -2 \; - \; \cancel{\lim\limits_{x \to -2} 3} \to 3}$

$= \dfrac{\cancel{\lim\limits_{x \to 2} 5} \to 5 \cdot \lim\limits_{x \to -2} x^3 \quad + 4}{-2 \quad -3}$

$= \dfrac{5 \cdot \left[\lim\limits_{x \to -2} x\right]^3 + 4}{-5} = \dfrac{5 \cdot (-2)^3 + 4}{-5}$

$= \dfrac{-40 + 4}{-5} = \dfrac{-36}{-5} = \boxed{\dfrac{36}{5}} = \boxed{7.2}$

Feb 15-9:02 AM

Evaluate $\lim_{x \to 0} \frac{\sqrt{x^2+4}}{3x-6}$

Direct plug in

$\frac{\sqrt{0^2+4}}{3(0)-6} = \frac{\sqrt{4}}{-6} = \frac{2}{-6} = \boxed{-\frac{1}{3}}$

$$\lim_{x \to 0} \frac{\sqrt{x^2+4}}{3x-6} = \frac{\lim_{x \to 0} \sqrt{x^2+4}}{\lim_{x \to 0} (3x-6)}$$

$$= \frac{\sqrt{\lim_{x \to 0} (x^2+4)}}{\lim_{x \to 0} 3x - \lim_{x \to 0} 6} = \frac{\sqrt{\lim_{x \to 0} x^2 + \lim_{x \to 0} 4}}{3 \lim_{x \to 0} x - \lim_{x \to 0} 6}$$

$$= \frac{\sqrt{(\lim_{x \to 0} x)^2 + \lim_{x \to 0} 4}}{3 \lim_{x \to 0} x - \lim_{x \to 0} 6} = \frac{\sqrt{0^2+4}}{3(0)-6}$$

$$= \frac{\sqrt{4}}{-6} = \frac{2}{-6} = \boxed{-\frac{1}{3}}$$

~~$-.3$~~

$-.\overline{3}$ ✓

Feb 15-9:07 AM

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$$

Is $f(x)$ continuous at $x=2$?

$$\lim_{x \to a} f(x) = f(a)$$

I.F.

1) $f(2) = 4$ ✓

2) $\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{0}{0}$

$= \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$ ✓

$= \lim_{x \to 2} (x+2) = 2+2 = \boxed{4}$

Since $\lim_{x \to 2} f(x) = f(2)$ then $f(x)$ is cont. at $x=2$.

Feb 15-9:13 AM

Evaluate $\lim_{x\to\infty} \frac{4x^2-8}{2x^2+4} = \frac{\infty}{\infty}$ I.F.

$$\lim_{x\to\infty} \frac{4x^2-8}{2x^2+4} = \lim_{x\to\infty} \frac{\frac{4x^2}{x^2}-\frac{8}{x^2}}{\frac{2x^2}{x^2}+\frac{4}{x^2}}$$

$$= \lim_{x\to\infty} \frac{4-\cancel{\frac{8}{x^2}}\to 0}{2+\cancel{\frac{4}{x^2}}\to 0} = \frac{4}{2} = \boxed{2}$$

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Evaluate $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x} = \frac{\sqrt{0^2+4}-2}{0} = \frac{\sqrt{4}-2}{0} = \frac{2-2}{0}$

$= \frac{0}{0}$ I.F.

$$\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x} = \lim_{x\to 0} \frac{(\sqrt{x^2+4}-2)(\sqrt{x^2+4}+2)}{x(\sqrt{x^2+4}+2)}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}\cancel{+4}\cancel{-4}}{\cancel{x}(\sqrt{x^2+4}+2)}$$

$$= \lim_{x\to 0} \frac{x}{\sqrt{x^2+4}+2} = \frac{0}{\sqrt{0+4}+2}$$

$$= \frac{0}{4}$$

$$= \boxed{0}$$

Do not use Ø for 0.

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Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+4x}-x\right) = \infty - \infty$

I.F. $\frac{0}{0}, \frac{\infty}{\infty}$

Let $x = 1000$

$\sqrt{1000^2+4(1000)} - 1000 \approx 1.998 \approx \boxed{2}$

$$\lim_{x\to\infty}\left(\sqrt{x^2+4x}-x\right) = \lim_{x\to\infty}\frac{(\sqrt{x^2+4x}-x)(\sqrt{x^2+4x}+x)}{\sqrt{x^2+4x}+x}$$

$(A+B)(A-B) = A^2 - B^2$

$$= \lim_{x\to\infty}\frac{\cancel{x^2}+4x-\cancel{x^2}}{\sqrt{x^2+4x}+x}$$

$$= \lim_{x\to\infty}\frac{4x}{\sqrt{x^2+4x}+x} = \frac{\infty}{\infty}$$

Divide everything by highest power of x

$$= \lim_{x\to\infty}\frac{\frac{4x}{x}}{\frac{\sqrt{x^2+4x}}{x}+\frac{x}{x}}$$

$$= \lim_{x\to\infty}\frac{4}{\sqrt{\frac{x^2+4x}{x^2}}+1}$$

$$= \lim_{x\to\infty}\frac{4}{\sqrt{1+\cancel{\frac{4}{x}}^{\to 0}}+1}$$

$$= \frac{4}{1+1} = \frac{4}{2} = \boxed{2}$$

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Class QZ 5 Box Your Final Ans

1) $\lim_{x\to 2}\frac{x^3-8}{x-2} = \frac{0}{0}$ I.F. $\lim_{x\to 2}\frac{x^3-8}{x-2} = \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x-2}$

$= \lim_{x\to 2}(x^2+2x+4)$

$= \boxed{12}$

2) $\lim_{x\to\infty}\frac{2x+5}{x-1} = \frac{\infty}{\infty}$ I.F. $\lim_{x\to\infty}\frac{2x+5}{x-1} = \lim_{x\to\infty}\frac{2+\frac{5}{x}}{1-\frac{1}{x}} = \frac{2}{1} = \boxed{2}$

3) $\lim_{x\to 16}\frac{\sqrt{x}-4}{x-16} = \frac{0}{0}$ I.F. $\lim_{x\to 16}\frac{\sqrt{x}-4}{x-16} = \lim_{x\to 16}\frac{(\sqrt{x}-4)(\sqrt{x}+4)}{(x-16)(\sqrt{x}+4)}$

$= \lim_{x\to 16}\frac{x-16}{(x-16)(\sqrt{x}+4)} = \lim_{x\to 16}\frac{1}{\sqrt{x}+4}$

$= \frac{1}{4+4}$

$= \boxed{\frac{1}{8}}$

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