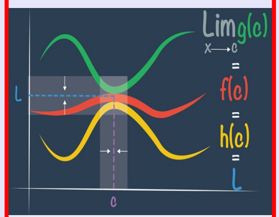
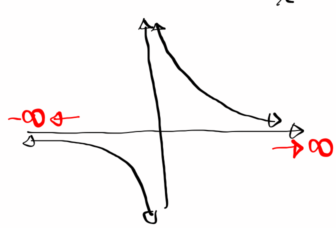
Calculus I

Lecture 7



Feb 19-8:47 AM

Graph $f(x) = \frac{1}{x}$



$\lim_{x \to \infty} f(x) = \boxed{0}$

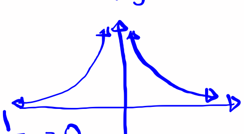
$\lim_{x \to -\infty} f(x) = \boxed{0}$

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\lim_{x \to -\infty} \frac{1}{x} = 0$

if n is a positive even integer

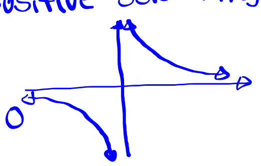
Graph of $\frac{1}{x^n}$



$\lim_{x \to \infty} \frac{1}{x^n} = 0$, $\lim_{x \to -\infty} \frac{1}{x^n} = 0$

if n is a positive odd integer

Graph of $\frac{1}{x^n}$



$\lim_{x \to \infty} \frac{1}{x^n} = 0$

$\lim_{x \to -\infty} \frac{1}{x^n} = 0$

Feb 13-9:53 AM

Evaluate $\lim_{x\to\infty} \frac{2x-1}{5x+3} = \frac{\infty}{\infty}$ I.F.

Divide numerator and denominator by the highest power of x.

$$\lim_{x\to\infty} \frac{2x-1}{5x+3} = \lim_{x\to\infty} \frac{\frac{2x}{x} - \frac{1}{x}}{\frac{5x}{x} + \frac{3}{x}} = \lim_{x\to\infty} \frac{2 - \cancel{\frac{1}{x}}^{\to 0}}{5 + \cancel{\frac{3}{x}}^{\to 0}} = \boxed{\frac{2}{5}} = \boxed{.4}$$

evaluate $\frac{2x-1}{5x+3}$ for $x = 1000$

$$\frac{2(1000)-1}{5(1000)+3} = \frac{1999}{5003} = .3995 \approx \boxed{.4}$$

Now try 10000

$$\frac{2(10000)-1}{5(10000)+3} = \frac{19999}{50003} = .39995 \approx \boxed{.4}$$

Feb 14-8:50 AM

Evaluate $\lim_{x\to-\infty} \frac{4x^2+x}{5x^3-1} = \frac{\infty}{-\infty}$ I.F.

Divide numerator and denominator by the highest power of x

$$\lim_{x\to-\infty} \frac{4x^2+x}{5x^3-1} = \lim_{x\to-\infty} \frac{\frac{4x^2}{x^3} + \frac{x}{x^3}}{\frac{5x^3}{x^3} - \frac{1}{x^3}} = \lim_{x\to-\infty} \frac{\cancel{\frac{4}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0}}{5 - \cancel{\frac{1}{x^3}}^{\to 0}}$$

$$= \frac{0}{5} = \boxed{0}$$

Evaluate $\frac{4x^2+x}{5x^3-1}$ for $x = -1000$

$$\frac{4(-1000)^2 + (-1000)}{5(-1000)^3 - 1} \approx -.0007 \approx \boxed{0}$$

Feb 14-8:59 AM

Evaluate

$$\lim_{x\to\infty} \frac{5x^2+7}{3x^2+x} = \frac{\infty}{\infty} \quad \text{I.F.}$$

Divide everything by x^2

$$\lim_{x\to\infty} \frac{5x^2+7}{3x^2+x} = \lim_{x\to\infty} \frac{5+\cancel{\frac{7}{x^2}}^{\to 0}}{3+\cancel{\frac{1}{x}}^{\to 0}} = \boxed{\frac{5}{3}}$$

$$\lim_{x\to-\infty} \frac{2-x}{4x-1} = \frac{\infty}{-\infty} \quad \text{I.F.}$$

$$\lim_{x\to-\infty} \frac{2-x}{4x-1} = \lim_{x\to-\infty} \frac{\cancel{\frac{2}{x}}^{\to 0}-1}{4-\cancel{\frac{1}{x}}^{\to 0}} = \boxed{\frac{-1}{4}} = \boxed{-.25}$$

Divided everything by x, and Simplified

Feb 14-9:07 AM

Intro. to Continuity:

$f(x)$ is continuous at $x=a$ if

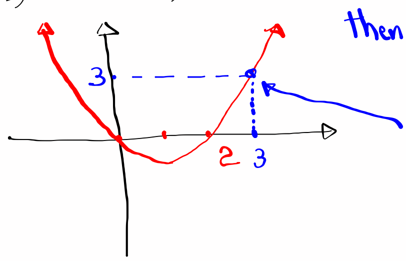
$$\lim_{x\to a} f(x) = f(a)$$

ex: Is $f(x) = x^2-2x$ cont. at $x=3$? $\quad f(x) = x(x-2)$

$$\lim_{x\to 3} f(x) = \lim_{x\to 3}(x^2-2x) = 3^2-2(3) = 3$$

$$f(3) = 3^2-2(3) = 3$$

Since $\lim_{x\to 3} f(x) = f(3)$ then $f(x)$ is Cont. at $x=3$.



No gap
No jump
No hole

Feb 14-9:13 AM

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$$

Is $f(x)$ cont. at $x=0$?

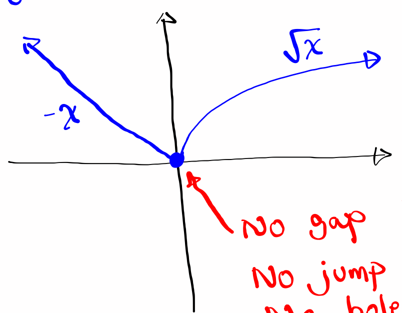
$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x) = 0$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sqrt{x} = 0$$

$f(0) = \sqrt{0} = 0$ ✓

$\lim_{x\to 0} f(x) = 0$ ✓

Since $\lim_{x\to 0} f(x) = f(0)$ then $f(x)$ is cont. at $x=0$.



No gap
No jump
No hole

} Cont. at $x=0$

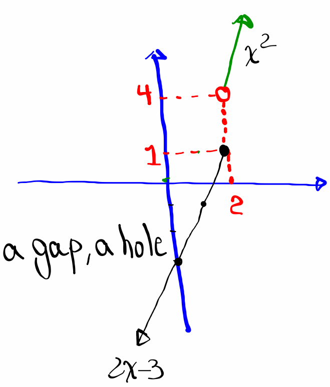
Feb 14-9:19 AM

$$f(x) = \begin{cases} 2x-3 & \text{if } x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$$

Is $f(x)$ cont. at $x=2$?

at $x=2$, we have a jump, a gap, a hole

Not cont. at $x=2$.



$f(2) = 2(2) - 3 = 1$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x-3) = 1$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2 = 4$$

$\lim_{x\to 2} f(x) =$ D.N.E.

Since one-sided limits are not equal

For cont. at $x=2$ $\lim_{x\to 2} f(x) = f(2)$

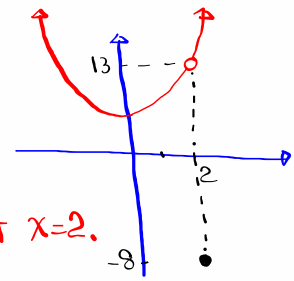
D.N.E. ✓

Not Cont. at $x=2$.

Feb 14-9:24 AM

Your turn:

$$f(x) = \begin{cases} 3x^2 + 1 & \text{if } x \neq 2 \\ -8 & \text{if } x = 2 \end{cases}$$



Is $f(x)$ cont. at $x=2$?

NO, we have a hole at $x=2$.

$\lim_{x \to 2} f(x) = 13$, $f(2) = -8$

$\lim_{x \to 2} f(x) \neq f(2) \Rightarrow f(x)$ is not continuous at $x=2$.

Polynomial Functions are continous everywhere.

Rational functions are continous every where except where deno. = 0.

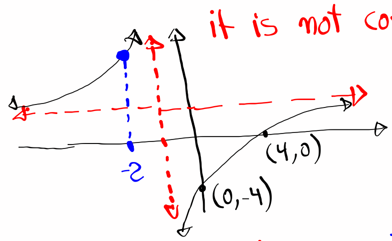
Feb 14-9:33 AM

Is $f(x) = x^3 - 5x^2 + 4$ cont. at $x = -2$?

Yes, $f(x)$ is a polynomial function.

Is $f(x) = \dfrac{x-4}{x+1}$ Cont. at $x = -2$?

Yes, $f(x)$ is a rational function, it is not cont. at $x = -1$.



V.A. $x = -1$

H.A. $y = 1$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x-4}{x+1} = \lim_{x \to \infty} \frac{1 - \frac{4}{x} \to 0}{1 + \frac{1}{x} \to 0} = \boxed{1}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x-4}{x+1} = \lim_{x \to -\infty} \frac{1 - \frac{4}{x} \to 0}{1 + \frac{1}{x} \to 0} = \boxed{1}$$

Feb 14-9:41 AM

Find all x values of discontinuity

1) $f(x) = x^2 - x - 6$

None, $f(x)$ is Polynomial $\Rightarrow$ Cont. $(-\infty, \infty)$

2) $f(x) = \dfrac{x}{x^2 - x - 6}$

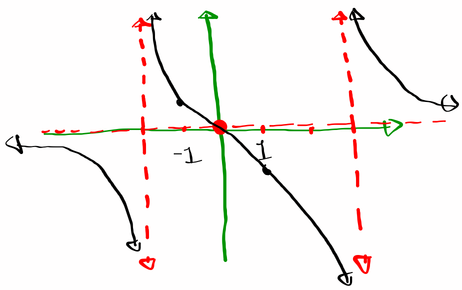
discontinuity happens when deno. $= 0$.

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ $\quad$ $x = -2$

discontinuous at $x = -2$ and $x = 3$



$f(1) = \dfrac{1}{-6}$

$f(-1) = \dfrac{-1}{-4} = \dfrac{1}{4}$

Feb 14-9:50 AM