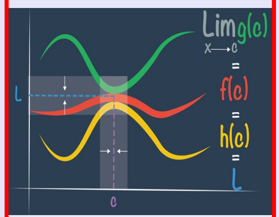
Calculus I

Lecture 6



Class QZ 3

$\boxed{\text{Box Your Final Ans.}}$

Evaluate $\lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4} = \frac{(-1)^2+6(-1)+5}{(-1)^2-3(-1)-4} = \frac{1-6+5}{1+3-4} = \frac{0}{0}$

✓ I.F.

$$\lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4} = \lim_{x \to -1} \frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x-4)} = \lim_{x \to -1} \frac{x+5}{x-4} = \frac{-1+5}{-1-4}$$

$$= \frac{4}{-5} = \boxed{-\frac{4}{5}} \checkmark$$

$$\cancel{\lim_{x \to -1} -\frac{4}{5}}$$

Class QZ 4

Factor Completely

$A^2 - B^2 = (A+B)(A-B)$

1) $x^3 - 64x = x(x^2 - 64) = \boxed{x(x+8)(x-8)}$ ✓

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

2) $2x^3 + 250 = 2(x^3 + 125) = \boxed{2(x+5)(x^2 - 5x + 25)}$ ✓

Feb 13-7:34 AM

1) Evaluate $\lim_{x \to 5} \sqrt{x^3 - 5x} = \sqrt{5^3 - 5(5)} = \sqrt{125 - 25}$

$= \sqrt{100} = \boxed{10}$ ✓

2) Evaluate $\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^2 + x - 6} = \frac{2^2 - 4(2) + 4}{2^2 + 2 - 6} = \frac{0}{0}$ I.F.

$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^2 + x - 6} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x-2)}{\cancel{(x-2)}(x+3)} = \lim_{x \to 2} \frac{x-2}{x+3} = \frac{2-2}{2+3}$

$= \frac{0}{5} = \boxed{0}$

Feb 13-9:03 AM

$$\lim_{x\to 0^+} \frac{x}{|x|} = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = \boxed{1}$$

$$|x| = \begin{cases} -x & \text{if } x<0 \\ x & \text{if } x\geq 0 \end{cases}$$

0 x>0

$$\lim_{x\to 0} \frac{x}{|x|} \quad \text{D.N.E.}$$

$$\lim_{x\to 0^-} \frac{x}{|x|} = \lim_{x\to 0^-} \frac{x}{-x} = \lim_{x\to 0^-} (-1) = \boxed{-1}$$

Since

$$\lim_{x\to 0^+} \frac{x}{|x|} \neq \lim_{x\to 0^-} \frac{x}{|x|}$$

x<0 0

Feb 13-9:10 AM

1) Evaluate $\lim_{x\to 2} \frac{x-2}{x^3-8} = \frac{2-2}{2^3-8} = \frac{0}{0}$ I.F.

$$\lim_{x\to 2} \frac{x-2}{x^3-8} = \lim_{x\to 2} \frac{\cancel{x-2}}{(\cancel{x-2})(x^2+2x+4)} = \lim_{x\to 2} \frac{1}{x^2+2x+4} = \boxed{\frac{1}{12}}$$

2) Evaluate $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{4-4}{\sqrt{4}-2} = \frac{0}{0}$ I.F.

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4} \frac{(\cancel{x-4})(\sqrt{x}+2)}{\underbrace{(\sqrt{x}-2)(\sqrt{x}+2)}_{\cancel{x-4}}} = \lim_{x\to 4} (\sqrt{x}+2)$$

$$= \sqrt{4}+2 = \boxed{4} \checkmark$$

FOIL & Simplify

Feb 13-9:17 AM

Given $f(t) = \begin{cases} t^2 - 4 & \text{if } t < 2 \\ \sqrt{t-2} + 3 & \text{if } t \geq 2 \end{cases}$ Piece-wise function.

1) $\lim_{t \to 2^-} f(t) = \lim_{t \to 2^-} (t^2 - 4)$
$= 2^2 - 4 = \boxed{0}$

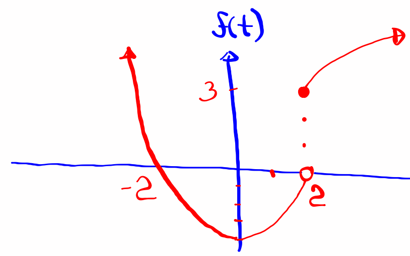
$t < 2 \quad 2$

2) $\lim_{t \to 2^+} f(t) = \lim_{t \to 2^+} (\sqrt{t-2} + 3)$
$= \sqrt{2-2} + 3 = \boxed{3}$

$2 \quad t > 0$

3) $\lim_{t \to 2} f(t) = \text{D.N.E.}$

4) $f(2) = \sqrt{2 - 2} + 3$
$= \boxed{3}$



Feb 13-9:24 AM

Given $f(x) = \begin{cases} \dfrac{x^2 - 9}{x + 3} & x \neq -3 \\ K & x = -3 \end{cases}$

Find K Such that $f(-3) = \lim_{x \to -3} f(x)$

$K = \lim_{x \to -3} \dfrac{x^2 - 9}{x + 3}$ After plug in $\frac{0}{0}$ I.F

$K = \lim_{x \to -3} \dfrac{(x+3)(x-3)}{x+3} = \lim_{x \to -3} (x - 3) = -3 - 3 = \boxed{-6}$

$\boxed{K = -6}$

Feb 13-9:36 AM

Given $f(x) = \frac{x-1}{x+1}$

$x=2 \to \frac{1}{3}$

Evaluate $\lim\limits_{x\to 2} \frac{f(x)-\frac{1}{3}}{x-2} = \lim\limits_{x\to 2} \frac{\frac{x-1}{x+1}-\frac{1}{3}}{2-2} = \frac{0}{0}$ I.F.

$$\lim_{x\to 2} \frac{f(x)-\frac{1}{3}}{x-2} = \lim_{x\to 2} \frac{\frac{x-1}{x+1}-\frac{1}{3}}{x-2}$$

$\boxed{LCD = 3(x+1)}$

$$= \lim_{x\to 2} \frac{\cancel{3(x+1)}\cdot\frac{x-1}{\cancel{x+1}} - \cancel{3}(x+1)\cdot\frac{1}{\cancel{3}}}{3(x+1)(x-2)} = \lim_{x\to 2} \frac{3(x-1)-(x+1)}{3(x+1)(x-2)}$$

$$= \lim_{x\to 2} \frac{3x-3-x-1}{3(x+1)(x-2)} = \lim_{x\to 2} \frac{\overset{2(\cancel{x-2})}{2x-4}}{3(x+1)\cancel{(x-2)}}$$

$$= \lim_{x\to 2} \frac{2}{3(x+1)} = \boxed{\frac{2}{9}}$$

Feb 13-9:40 AM

Given $f(x) = \frac{1}{x^2}$

Evaluate $\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{x^2\cancel{(x+h)^2}\cdot\frac{1}{\cancel{(x+h)^2}} - \cancel{x^2}(x+h)^2\cdot\frac{1}{\cancel{x^2}}}{h\,x^2(x+h)^2}$$

$LCD = x^2(x+h)^2$

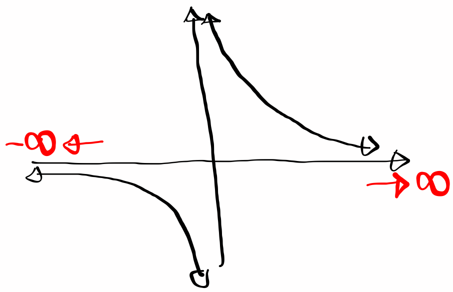
$$= \lim_{h\to 0} \frac{x^2-(x+h)^2}{h\,x^2(x+h)^2} = \lim_{h\to 0} \frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{h\,x^2(x+h)^2}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}\,x^2(x+h)^2} = \frac{-2x-0}{x^2(x+0)^2} = \frac{-2\cancel{x}}{x^2\cdot \cancel{x^2}}$$

$$= \boxed{\frac{-2}{x^3}}$$

Feb 13-9:47 AM

Graph $f(x) = \frac{1}{x}$



$$\lim_{x \to \infty} f(x) = \boxed{0}$$

$$\lim_{x \to -\infty} f(x) = \boxed{0}$$

Feb 13-9:53 AM