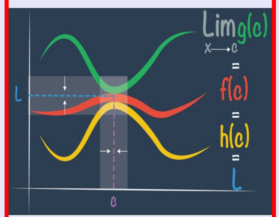
Calculus I

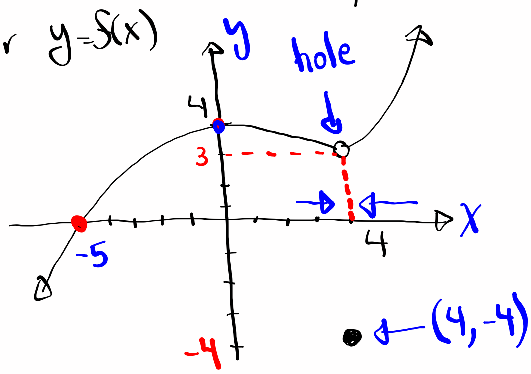
Lecture 5



Feb 19-8:47 AM

Class QZ 2

Consider the graph below for $y=f(x)$



1) Y-Int $(0,4)$ ✓

2) x-Int $(-5,0)$ ✓

3) as $x \to 4^+$, $y \to 3$ ✓

4) as $x \to 4^-$, $y \to 3$ ✓

5) $f(4) = -4$ ✓

Feb 8-9:43 AM

Intro. to limits:

If the value of $f(x)$ approaches the number L_1 as x approaches a from the right side

$$\lim_{x \to a^+} f(x) = L_1 \qquad \text{one-sided limit}$$

If the value of $f(x)$ approaches the number L_2 as x approaches a from the left side

$$\lim_{x \to a^-} F(x) = L_2 \qquad \text{one-sided limit}$$

If $L_1 = L_2$

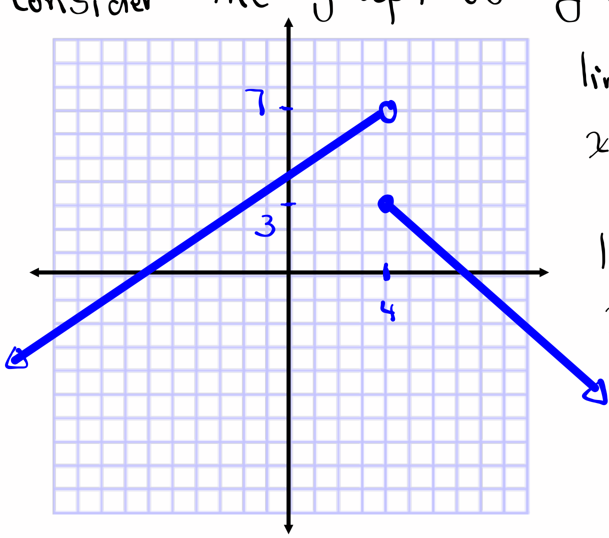
$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$$

then

$$\lim_{x \to a} f(x) = L \qquad \text{Two-sided limit}$$

Feb 12-8:48 AM

Consider the graph of $y = f(x)$ below



$$\lim_{x \to 4^+} f(x) = 3$$

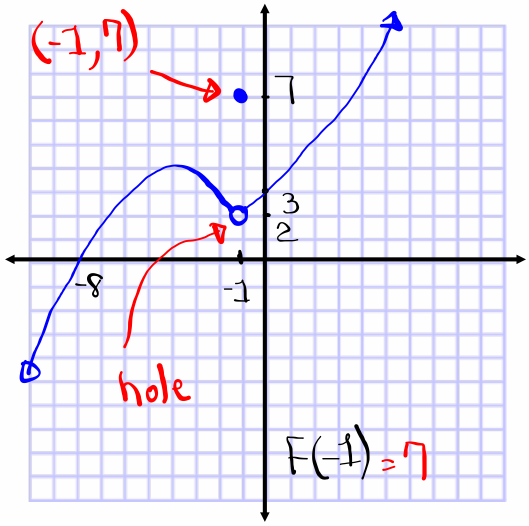
$$\lim_{x \to 4^-} f(x) = 7$$

$\lim_{x \to 4} F(x)$ Does not exist.

Since $\lim_{x \to 4^+} f(x) \neq \lim_{x \to 4^-} f(x)$

Feb 12-8:53 AM

Consider the graph of $y = f(x)$ below



$$\lim_{x \to -1^-} f(x) = 2$$

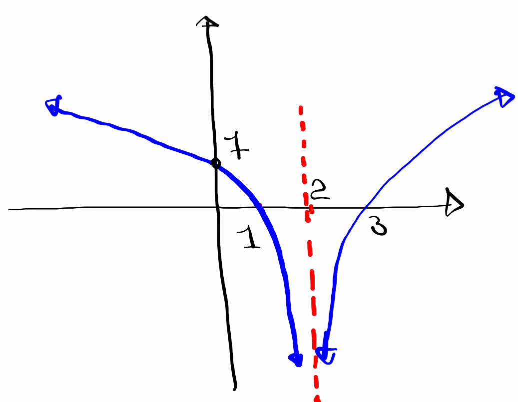
$$\lim_{x \to -1^+} f(x) = 2$$

Since $\lim_{x \to -1^-} F(x) = \lim_{x \to -1^+} f(x)$

then $\lim_{x \to -1} f(x) = 2$

Feb 12-8:57 AM

Consider the graph of $y = f(x)$ below



$x = 2$ Vertical Asymptote

$f(2)$ is undefined

Y-Int $(0,1)$

x-Int $(1,0), (3,0)$

$\lim_{x \to 2^+} f(x) = -\infty$ $\lim_{x \to 2^-} f(x) = -\infty$ $\lim_{x \to 2} f(x) = -\infty$

Feb 12-9:01 AM

How to compute limits:

1) Plug it in, and simplify.

$$\lim_{x\to 4}\left(x^3-\sqrt{x}\right) = 4^3-\sqrt{4} = 64-2 = \boxed{62}$$

$$\lim_{x\to 0}\frac{x^3-8}{x-2} = \frac{0^3-8}{0-2} = \frac{-8}{-2} = \boxed{4}$$

$$\lim_{x\to 2}\frac{x^3-8}{x-2} = \frac{2^3-8}{2-2} = \frac{8-8}{2-2} = \frac{0}{0}$$

Indeterminate form

2) If we have an I.F., use algebra to simplify

$$\lim_{x\to 2}\frac{x^3-8}{x-2} = \lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$$

$$= \lim_{x\to 2}(x^2+2x+4) = 2^2+2(2)+4 = \boxed{12}$$

Please review your factoring

A^2-B^2 A^2+B^2

A^3-B^3 A^3+B^3

Feb 12-9:07 AM

Evaluate $\lim_{x\to 0}\frac{x^3-25x}{x^2+5x} = \frac{0^3-25(0)}{0^2+5(0)} = \frac{0}{0}$ I.F.

$$\lim_{x\to 0}\frac{x^3-25x}{x^2+5x} = \lim_{x\to 0}\frac{\cancel{x}(x^2-25)}{\cancel{x}(x+5)} = \frac{0^2-25}{0+5}$$

$$= \frac{-25}{5} = \boxed{-5}$$

$$\lim_{x\to 5}\frac{x^3-25x}{x^2+5x} = \frac{5^3-25(5)}{5^2+5(5)} = \frac{125-125}{25+25} = \frac{0}{50}$$

$$= \boxed{0}$$

$$\lim_{x\to -5}\frac{x^3-25x}{x^2+5x} = \frac{(-5)^3-25(-5)}{(-5)^2+5(-5)} = \frac{-125+125}{25-25} = \frac{0}{0} \text{ I.F.}$$

$$\lim_{x\to -5}\frac{x^3-25x}{x^2+5x} = \lim_{x\to -5}\frac{\cancel{x}\cancel{(x+5)}(x-5)}{\cancel{x}\cancel{(x+5)}} = \lim_{x\to -5}(x-5)$$

$$= -5-5 = \boxed{-10}$$

Feb 12-9:16 AM

Evaluate $\lim_{x\to 1} \dfrac{\frac{1}{x} - 1}{x-1} = \dfrac{\frac{1}{1}-1}{1-1} = \dfrac{0}{0}$ I.F.

LCD $= x$

$$\lim_{x\to 1} \frac{\frac{1}{x} - 1}{x-1} = \lim_{x\to 1} \frac{x\cdot\frac{1}{x} - x\cdot 1}{x(x-1)}$$

$$\boxed{\frac{a-b}{b-a} = -1}$$

$$= \lim_{x\to 1} \frac{\overset{-1}{\cancel{1-x}}}{x\,\cancel{(x-1)}} = \lim_{x\to 1} \frac{-1}{x} = \frac{-1}{1}$$

$$= \boxed{-1}$$

Feb 12-9:25 AM

Given $f(x) = x^2 - 5x$ $\quad f(3) = 3^2 - 5(3)$

$= 9 - 15$

$= -6$

Evaluate $\lim_{x\to 3} \dfrac{f(x) - f(3)}{x-3}$

$$\lim_{x\to 3} \frac{f(x) - f(3)}{x-3} = \lim_{x\to 3} \frac{x^2 - 5x - (-6)}{x-3}$$

$$= \lim_{x\to 3} \frac{x^2 - 5x + 6}{x-3} = \frac{3^2 - 5(3) + 6}{3-3}$$

$$= \frac{0}{0} \text{ I.F.}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x-2)}{\cancel{x-3}}$$

$$= \lim_{x\to 3} (x-2) = 3 - 2 = \boxed{1}$$

Feb 12-9:29 AM

Given $f(x)=\sqrt{x}$

Evaluate $\lim_{x\to 9} \frac{f(x)-f(9)}{x-9} = \lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \frac{\sqrt{9}-3}{9-9} = \frac{3-3}{0} = \frac{0}{0}$

I.F.

Rationalize the numerator

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to 9} \frac{(\sqrt{x})^2-(3)^2}{(x-9)(\sqrt{x}+3)}$$

Review

$(A-B)(A+B)=A^2-B^2$

$$= \lim_{x\to 9} \frac{\cancel{x-9}}{(\cancel{x-9})(\sqrt{x}+3)}$$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$$

Feb 12-9:35 AM

Class QZ 3

Box Your Final Ans.

Evaluate $\lim_{x\to -1} \frac{x^2+6x+5}{x^2-3x-4} = \frac{(-1)^2+6(-1)+5}{(-1)^2-3(-1)-4} = \frac{1-6+5}{1+3-4} = \frac{0}{0}$

I.F.

$$\lim_{x\to -1} \frac{x^2+6x+5}{x^2-3x-4} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x-4)} = \lim_{x\to -1} \frac{x+5}{x-4} = \frac{-1+5}{-1-4}$$

$$= \frac{4}{-5} = \boxed{-\frac{4}{5}}$$

Feb 12-9:46 AM