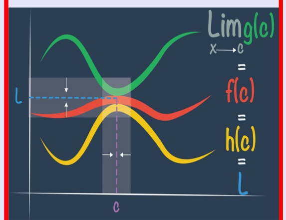
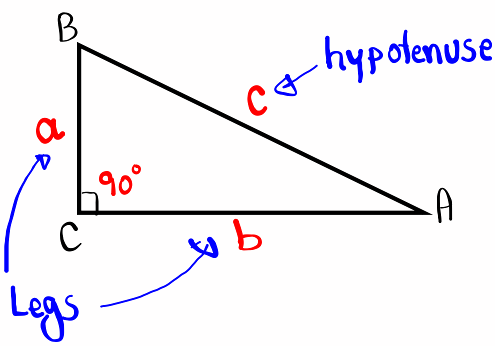
Calculus I

Lecture 4





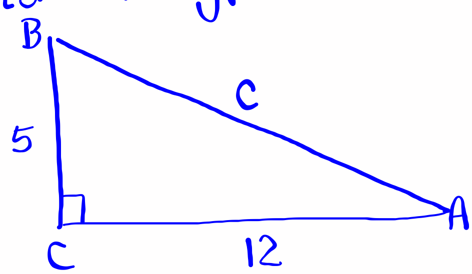
Right Triangle

$A + B = 90^\circ$

$$\boxed{a^2 + b^2 = c^2}$$

Pythagorean Thrm

Find the hypotenuse.



$a^2 + b^2 = c^2$

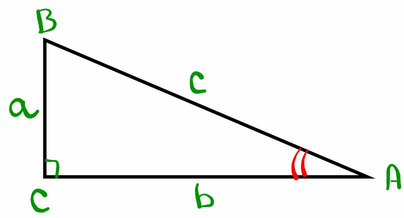
$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$c^2 = 169$

$c = \sqrt{169}$

$$\boxed{c = 13}$$



$\sin A = \frac{\text{opp.}}{\text{hyp.}} = \frac{a}{c}$

$\cos A = \frac{\text{Adj.}}{\text{hyp.}} = \frac{b}{c}$

$\tan A = \frac{\text{opp.}}{\text{Adj.}} = \frac{a}{b}$

$\csc A = \frac{1}{\sin A} = \frac{c}{a}$

$\sec A = \frac{1}{\cos A} = \frac{c}{b}$

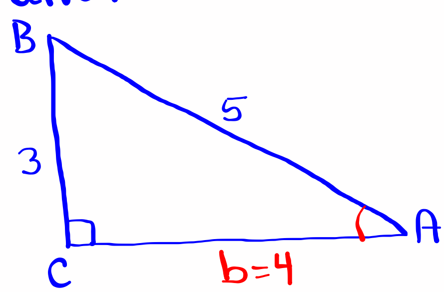
$\cot A = \frac{1}{\tan A} = \frac{b}{a}$

Prove $1 + \tan^2 A = \sec^2 A$ ✓

$$1 + \tan^2 A = 1 + \left(\frac{a}{b}\right)^2 = 1 + \frac{a^2}{b^2} = \frac{b^2}{b^2} + \frac{a^2}{b^2}$$

$$= \frac{b^2 + a^2}{b^2} = \frac{c^2}{b^2} = \left(\frac{c}{b}\right)^2 = \boxed{\sec^2 A}$$

Given



$3^2 + b^2 = 5^2$

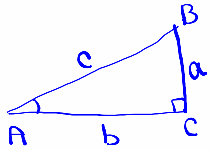
⋮

$b = 4$

Complete the chart below

$\sin A = \frac{3}{5}$	$\csc A = \frac{5}{3}$
$\cos A = \frac{4}{5}$	$\sec A = \frac{5}{4}$
$\tan A = \frac{3}{4}$	$\cot A = \frac{4}{3}$

Prove $\tan A = \frac{\sin A}{\cos A}$

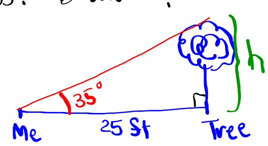


$\tan A = \frac{a}{b}$

Divide both numerator and deno. by c

$= \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{\sin A}{\cos A}$ ✓

I am standing $\boxed{25 \text{ ft}}$ from a tree. From my feet, the angle of elevation to the top of the tree is 35°. Draw & find the height of the tree.



$\tan 35° = \frac{h}{25}$

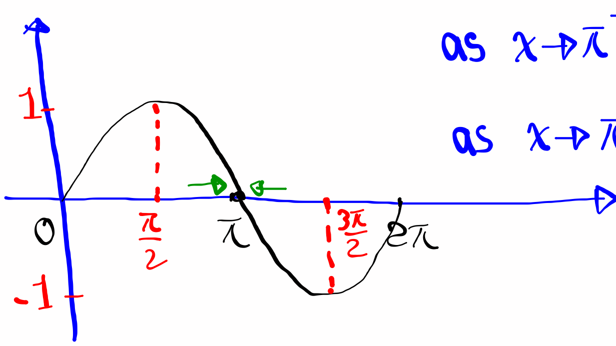
Cross-multiply

$h = 25 \cdot \tan 35°$

$h \approx 17.5$

about 17.5 ft

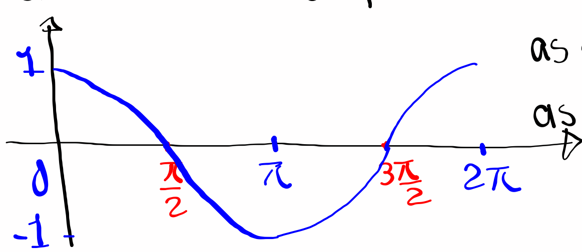
Below is the graph of one period of $y = \sin x$.



as $x \to \pi^+$, $y \to 0$

as $x \to \pi^-$, $y \to 0$

Below is the graph of one period of $y = \cos x$.



as $x \to \pi^+$, $y \to -1$

as $x \to \pi^-$, $y \to -1$

I want to evaluate $\frac{\sin x}{x}$ at $x=0$.

Plug it in $\rightarrow$ $\frac{\sin 0}{0} = \frac{0}{0}$ Indeterminate

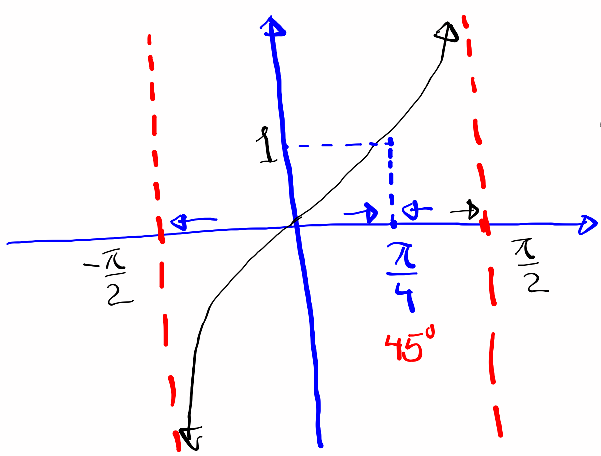
Let's evaluate at $x=.001$ $\quad \frac{\sin .001}{.001} = .999999833 \approx 1$

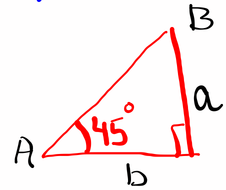
now do $x=-.001$ $\quad \frac{\sin(-.001)}{-.001} = .99999\ldots \approx 1$

as $x \rightarrow 0^+$, $\frac{\sin x}{x} \rightarrow 1$

as $x \rightarrow 0^-$, $\frac{\sin x}{x} \rightarrow 1$

Below is the graph of one period of $y=\tan x$





$a=b$

$\tan 45° = \frac{a}{b} = 1$

as $x \rightarrow \frac{\pi}{4}^+$, $y \rightarrow 1$

as $x \rightarrow \frac{\pi}{4}^-$, $y \rightarrow 1$

as $x \rightarrow \frac{\pi}{2}^-$, $y \rightarrow \infty$

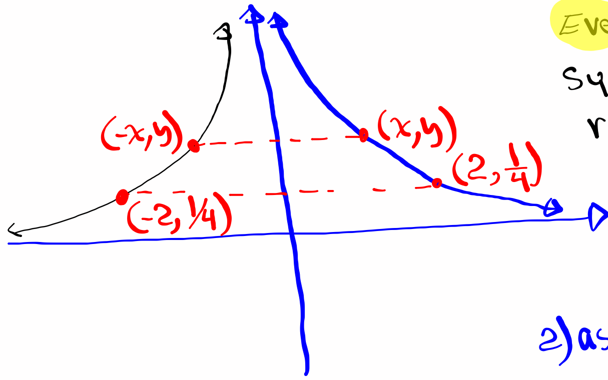
as $x \rightarrow -\frac{\pi}{2}^+$, $y \rightarrow -\infty$

Below is half of the graph of $f(x)=\frac{1}{x^2}$

$f(-x)=f(x)$

Even Function

symmetric with respect to Y-axis



1) Draw the other half.

2) as $x\to 0^+$, $y\to\infty$

3) as $x\to 0^-$, $y\to\infty$

4) as $x\to\infty$, $y\to 0$

5) as $x\to-\infty$, $y\to 0$

6) Domain $(-\infty,0)\cup(0,\infty)$

7) Range $(0,\infty)$

Find difference quotient for $f(x)=\frac{1}{x^2}$, evaluate the final result for $h=0$.

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

LCD: $(x+h)^2\cdot x^2$

$$=\frac{\cancel{(x+h)^2}\cdot x^2\cdot\frac{1}{\cancel{(x+h)^2}}-(x+h)^2\cdot\cancel{x^2}\cdot\frac{1}{\cancel{x^2}}}{(x+h)^2\cdot x^2\cdot h}$$

$(x+h)^2 \to x^2+2xh+h^2$

$$=\frac{x^2-(x+h)^2}{(x+h)^2\cdot x^2\cdot h}=\frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{(x+h)^2\cdot x^2\cdot h}$$

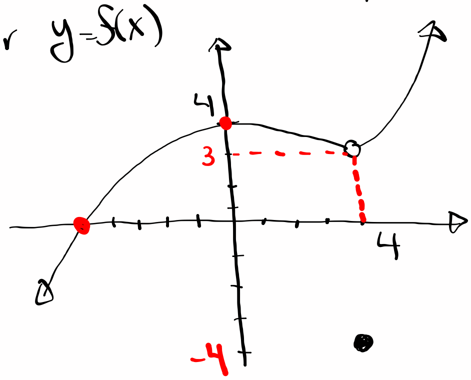
$$=\frac{\cancel{h}(-2x-h)}{(x+h)^2\cdot x^2\cdot\cancel{h}}=\frac{-2x-h}{(x+h)^2\cdot x^2}$$

For $h=0$

$$\frac{-2x-0}{(x+0)^2\cdot x^2}=\frac{-2\cancel{x}}{x^2\cdot x^{\cancel{2}}}=\boxed{\frac{-2}{x^3}}$$

Class QZ 2

Consider the graph below for $y = f(x)$



1) Y-Int (0,4)

2) x-Int (-5,0)

3) as $x \to 4^{+}$, $y \to 3$

4) as $x \to 4^{-}$, $y \to 3$

5) $f(4) = -4$