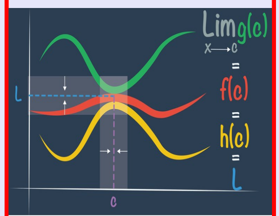
Calculus I

Lecture 34



Feb 19-8:47 AM

Class QZ 17:

Find all x-Values for all possible Critical points and inflection points for

$f(x) = x^4 - 2x^2 - 2.$

$f'(x) = 4x^3 - 4x \qquad f'(x) = 0 \to 4x(x^2 - 1) = 0 \quad \boxed{x = 0} \quad \boxed{x = \pm 1}$

$f''(x) = 12x^2 - 4 \qquad f''(x) = 0 \to 4(3x^2 - 1) = 0 \to \boxed{x = \pm\frac{\sqrt{3}}{3}}$

Find $\boxed{x}$ for C.P.

$\boxed{a)\ 0}$ b) $(0, -2)$

Apr 11-9:42 AM

$f(x) = (x+1)^5 - 5x + 2$

$f(0) = (0+1)^5 - 5(0) + 2 = 1 + 2 = 3 \rightarrow$ Y-Int $(0,3)$

$f(x)$ is a polynomial function, It is cont. & diff. on $(-\infty, \infty)$

$f'(x) = 5(x+1)^{5-1} \cdot 1 - 5 + 0$
$= 5(x+1)^4 - 5$

$f'(x) = 0$
$5(x+1)^4 - 5 = 0$
$5(x+1)^4 = 5$
$(x+1)^4 = 1$
$x + 1 = \pm\sqrt[4]{1}$
$x = -1 \pm 1$
$\boxed{x=-2}$ $\boxed{x=0}$

$f''(x) = 5 \cdot 4(x+1)^3 - 0$
$= 20(x+1)^3$

$f''(x) = 0 \rightarrow 20(x+1)^3 = 0$
$\boxed{x=-1}$

x	$-\infty$	-2	-1	0	∞
$f'(x)$	$+$	$-$	$-$	$+$	
$f''(x)$	$-$	$-$	$+$	$+$	
$f(x)$	Inc., CD	Dec., CD	Dec., CU	Inc. CU	

Concavity changed at -1, So $(-1, f(-1))$ is I.P. $(-1, 7)$

Inc. $(-\infty, -2), (0, \infty)$
Dec. $(-2, 0)$

C.U. $(-1, \infty)$
CD $(-\infty, -1)$

Make Sure to draw a rough graph.

Apr 15-8:48 AM

Verify the conditions of Rolle's Theorem, then find all number c that satisfy the conclusion of it for $f(x) = x^3 - x^2 - 6x + 2$ over $[0,3]$.

Since $f(x)$ is a Polynomial function, then

1) $f(x)$ is Cont. on $[0,3]$ ✓
2) $f(x)$ is diff. on $(0,3)$ ✓

Now $f(0) = f(3)$? ✓

$f(0) = 0^3 - 0^2 - 6(0) + 2 = \boxed{2}$
$f(3) = 3^3 - 3^2 - 6(3) + 2 = \boxed{2}$

There is at least a number c in $(0,3)$ such that $f'(c) = 0$.

$f'(x) = 3x^2 - 2x - 6 \rightarrow f'(c) = 3c^2 - 2c - 6 = 0$

Solve $3c^2 - 2c - 6 = 0$ using Q-Formula

$c = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$
$= \dfrac{2 \pm \sqrt{76}}{6}$

$\boxed{c_1 \approx 1.786 \text{ which is in } (0,3)}$

$c_2 \approx -1.120 \rightarrow$ is not in $(0,3)$

Apr 15-9:01 AM

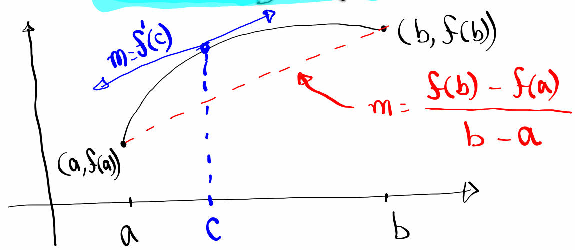
Mean Value Theorem

1) $\boxed{f(x) \text{ is cont. on } [a,b]}$

2) $\boxed{f(x) \text{ is diff. on } (a,b)}$

then there is at least a number C in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b-a}.$$



Tan. line at c is parallel to the secant line containing $(a, f(a))$ & $(b, f(b))$.

Apr 15-9:10 AM

$f(x) = x^3 - 3x + 2$, $[-2, 2]$

$f'(x) = 3x^2 - 3$

$f(2) = 2^3 - 3(2) + 2$
$= 8 - 6 + 2 = 4$

$f(-2) = (-2)^3 - 3(-2) + 2$
$= -8 + 6 + 2 = 0$

f(x) is a Polynomial Function,

1) f(x) is cont. → $[-2, 2]$

2) f(x) is diff. → $(-2, 2)$

by MVT, there is at least a number C in $(-2, 2)$ such that

$$f'(c) = \frac{f(2) - f(-2)}{2 - (-2)}$$

$$3c^2 - 3 = \frac{4 - 0}{4} \qquad 3c^2 - 3 = 1 \qquad 3c^2 = 4$$

$$c^2 = \frac{4}{3}$$

$$c = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$$

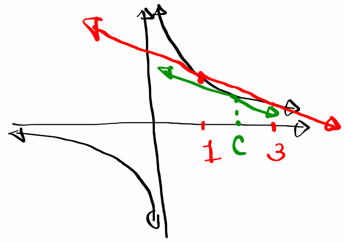
$c_1 \approx 1.155$ which is in $(-2, 2)$.

$c_2 \approx -1.155$ " " " $(-2, 2)$

Apr 15-9:17 AM

$f(x) = \frac{1}{x}$, $[1, 3]$

Reciprocal
function



Cont. everywhere except at 0.

Diff. 〃 〃 〃 0.

Conclusion of MVT

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$f'(x) = \frac{-1}{x^2}$

$\frac{-1}{c^2} = \frac{\frac{1}{3} - 1}{3 - 1}$

$\frac{-1}{c^2} = \frac{-2/3}{2}$

$\frac{1}{c^2} = \frac{1}{3}$

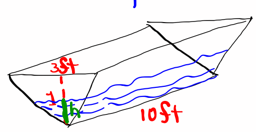
$c^2 = 3$

$c = \pm\sqrt{3}$

$c = \sqrt{3}$ is in $(1, 3)$

Apr 15-9:25 AM

A trough is 10 ft long and its ends have a shape of isosceles triangle with 3 ft at the top and a height of 1 f.



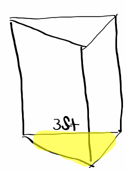
It is being filled with water at the rate of $12\ ft^3/min.$

How fast is water level rising $\frac{dh}{dt} = ?$

$\frac{dV}{dt} = 12\ ft^3/min.$

when water is 6 inches deep? $h = 6$ in.
$h = .5$ ft

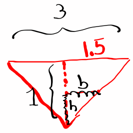
Volume



$V =$ Area of base $\cdot$ height

$= \frac{b\,h}{2} \cdot 10$

$= \frac{1.5h \cdot h}{2} \cdot 10$



$\frac{h}{b} = \frac{1}{1.5}$

$b = 1.5h$

$V = 7.5h^2$

$\frac{dV}{dt} = 7.5 \cdot \frac{d}{dt}[h^2]$

⋮

Make Sure to Finish

Apr 15-9:31 AM