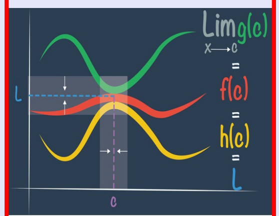
Calculus I

Lecture 32



Feb 19-8:47 AM

Critical Numbers:

C.N. are those values in the domain of $f(x)$ such that $f'(x)=0$ or $f'(x)$ is undefined.

ex: $f(x) = x^2 - 6x + 1$

Domain $(-\infty, \infty)$

$f'(x) = 2x - 6$

$f'(x) = 0$

$2x - 6 = 0$ $\boxed{x=3}$ C.N.

Critical Points are ordered-pairs of $(c, f(c))$ where c is a C.N.

$f(3) = 3^2 - 6(3) + 1$
$= 9 - 18 + 1$
$= -8$

$(3, f(3)) = \boxed{(3, -8)}$ C.P.

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Find all Critical numbers for $f(x)=\dfrac{x}{x-1}$.

$f(x)=\dfrac{x}{x-1}$

Rational Function

$x-1\neq 0$

$x\neq 1$

Domain $(-\infty,1)\cup(1,\infty)$

$f'(x)=\dfrac{1(x-1)-x\cdot 1}{(x-1)^2}$

$f'(x)=\dfrac{-1}{(x-1)^2}$

$f'(x)=0 \rightarrow -1=0$

No Solution

$f'(x)$ undefined

$(x-1)^2=0 \rightarrow x=1$

Not in the domain

$\therefore$ There are no C.N. or C.P.

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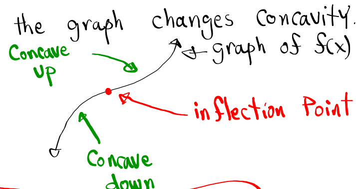
Inflection Points:

They are points in the domain of $f(x)$ where the graph changes concavity.

$f''(x)>0 \rightarrow$ C.U.

$f''(x)<0 \rightarrow$ C.D.



Possible inflection Points happen when $f''(x)=0$ or $f''(x)$ is undefined.

ex: $f(x)=x^4-4x^3$

Polynomial Function $\rightarrow$ Cont. everywhere

$f'(x)=4x^3-12x^2$

$f''(x)=12x^2-24x$

$f''(x)=0 \quad 12x(x-2)=0$

$x=0 \quad x=2$

Possible locations on the graph of $f(x)$ where Concavity changes.

x	$-\infty$	0	2	∞
$f''(x)$	+	−	+	
$f(x)$	C.U.	C.D.	C.U.	

Concavity changed at $x=0$ & $x=2$

Inflection Points $(0,f(0))$ & $(2,f(2))$

$(0,0)$, $(2,-16)$

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$f(x) = x^4 - 2x^2 + 3$

Polynomial function $\rightarrow$ Domain $(-\infty, \infty)$

$f'(x) = 4x^3 - 4x$

$f'(x) = 0 \rightarrow 4x(x^2 - 1) = 0 \rightarrow 4x(x+1)(x-1) = 0$

$\underbrace{x = 0,\ x = -1,\ x = 1}_{\text{C.N.}}$

C.P. $\begin{cases} (0, f(0)) = (0, 3) \\ (-1, f(-1)) = (-1, 2) \\ (1, f(1)) = (1, 2) \end{cases}$

$f''(x) = 12x^2 - 4$

$f''(x) = 0 \rightarrow 12x^2 - 4 = 0$

$4(3x^2 - 1) = 0$

$3x^2 - 1 = 0$

$x = \pm \frac{1}{\sqrt{3}}$

P.I.P. are at $x = \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}$

x	$-\infty$	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$	∞
$f''(x)$	+	−	+	
$f(x)$	C.U.	C.D.	C.U.	

I.P. are $\left(-\frac{\sqrt{3}}{3}, f\left(-\frac{\sqrt{3}}{3}\right)\right)$ & $\left(\frac{\sqrt{3}}{3}, f\left(\frac{\sqrt{3}}{3}\right)\right)$.

Concavity changes at $-\frac{\sqrt{3}}{3}$ and $\frac{\sqrt{3}}{3}$

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So

$$\boxed{\begin{array}{l} f'(x) > 0 \rightarrow f(x) \text{ is increasing} \\ f'(x) < 0 \rightarrow f(x) \text{ is decreasing} \\ f''(x) > 0 \rightarrow f(x) \text{ is concave up} \\ f''(x) < 0 \rightarrow f(x) \text{ is concave down} \end{array}}$$

$f(x) = x^2 - 6x + 3$

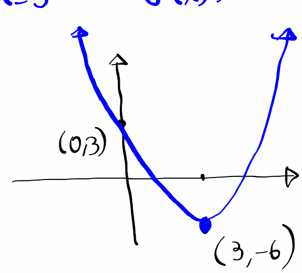
$f'(x) = 2x - 6$

$f'(x) = 0 \rightarrow x = 3$

$f''(x) = 2$

$f''(x) > 0$

x	$-\infty$	3	∞
$f'(x)$	−	+	
$f''(x)$	+	+	
$f(x)$	Decreasing, C.U.	Increasing, C.U.	



$(3, f(3)) = (3, -6)$

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$f(x) = \dfrac{x^2}{x-1}$

Domain $\to (-\infty, 1) \cup (1, \infty)$

Vertical Asy. at $x = 1$

x-Int & Y-Int $\to (0,0)$

$f'(x) = \dfrac{2x(x-1) - x^2 \cdot 1}{(x-1)^2}$

$f'(x) = \dfrac{x^2 - 2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$

$f'(x) = 0 \to x^2 - 2x = 0$

$x = 0, \ x = 2$

$f'(x)$ undefined $(x-1)^2 = 0$

$x = 1$

Not in the domain

Keep an eye on it.

$f''(x) = \dfrac{(2x-2)\cdot(x-1)^2 - (x^2-2x)\cdot 2(x-1)\cdot 1}{[(x-1)^2]^2}$

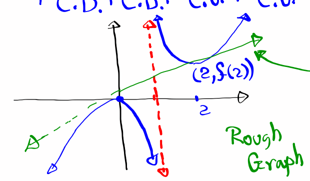
$= \dfrac{\cancel{(x-1)}[(2x-2)(x-1) - 2(x^2-2x)]}{(x-1)^4}$

$f''(x) = \dfrac{\cancel{2x^2} \cancel{-2x} \cancel{-2x} + 2 \cancel{-2x^2} \cancel{+4x}}{(x-1)^3} \Rightarrow f''(x) = \dfrac{2}{(x-1)^3}$

$f''(x) \neq 0$, $f''(x)$ undefined at $x = 1$

Not in the domain but Keep an eye on it.

x	$-\infty$ to 0	0 to 1	1 to 2	2 to ∞
$f'(x)$	+	−	−	+
$f''(x)$	−	−	+	+
$f(x)$	Inc. C.D.	Dec., C.D.	Dec., C.U.	Inc., C.U.



Slant Asymptote

Google this

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Rolle's theorem

1) $f(x)$ is Cont. on $[a,b]$ ✓

2) $f(x)$ is diff. on (a,b) ✓✓

3) $f(a) = f(b)$ ✓✓✓

Then there is a $\boxed{\text{at least one number } c}$ in (a,b) Such that $f'(c) = 0$

Please google Mean Value Theorem

ex: $f(x) = 3x^2 - 12x + 5 \quad [1,3]$

1) $f(x)$ is Cont. everywhere

2) $f(x)$ is diff. everywhere

3) $f(1) = 3(1)^2 - 12(1) + 5 = -4$

$f(3) = 3(3)^2 - 12(3) + 5 = -4$

$f(1) = f(3)$

$f'(x) = 6x - 12 \qquad f'(x) = 0 \qquad 6x - 12 = 0 \quad \boxed{x = 2}$

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