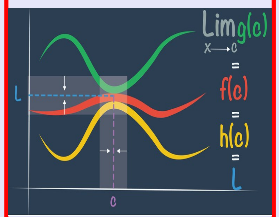
Calculus I

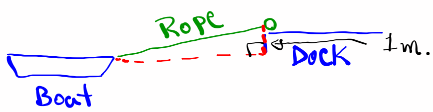
Lecture 31

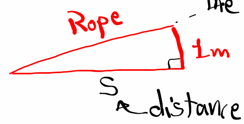


Feb 19-8:47 AM

A boat is pulled into a dock by a rope attached to the boat, and it is going through a pulley on the dock that is 1 m above the boat.

$\frac{dR}{dt} = -1$ m/s Pulling the rope





If the rope is pulled in at $\boxed{1 \text{ m/s}}$, How fast is the boat approaching the dock when it is 8 m from the dock.

$S^2 + 1^2 = R^2$

$8^2 + 1 = R^2 \to \boxed{R=}$

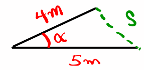
$S^2 + 1^2 = R^2$

$2S\frac{dS}{dt} + 0 = 2R\frac{dR}{dt}$

Solve $\boxed{\frac{dS}{dt} = ?}$

Apr 9-8:47 AM

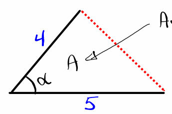
Two sides of a triangle are 4m and 5m.



s is changing

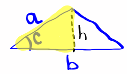
The angle between them is increasing at the rate of .06 rad/s.

At what rate is the area of the triangle changing when the angle between the two given side is $60°$?



Area is changing. $\frac{dA}{dt}$ when $\alpha = 60°$

How can we find area of a triangle when two sides and the angle between them is given.



$A = \frac{1}{2} bh$

$\sin C = \frac{h}{a} \rightarrow h = a \sin C$

$\boxed{A = \frac{1}{2} ab \sin C}$

$\text{Area} = \frac{1}{2} \cdot 4 \cdot 5 \cdot \sin\alpha$

$A = 10 \sin\alpha$

$\frac{dA}{dt} = 10 \cdot \cos\alpha \cdot \frac{d\alpha}{dt}$

$\frac{dA}{dt} = 10 \cdot \cos 60° \cdot (.06)$

$= .6 \cdot \frac{1}{2} = \boxed{.3 \ m^2/s}$

unit

Apr 9-8:56 AM

How to find abs. Max & abs. Min values of a cont. function $f(x)$ on $[a,b]$:

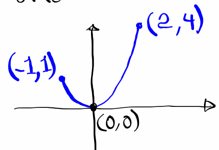
1) Always verify that $f(x)$ is cont. on $[a,b]$.

2) Find $f(a)$ & $f(b)$

3) Find all $\boxed{\text{critical points}}$ in (a,b). → $f'(x) = 0$ or undefined

4) Abs. Max & Abs. Min. are the largest and smallest values from steps 2 & 3.

Find abs. Max & abs. Min for $f(x) = x^2$ over $[-1,2]$



Abs. Max is $\boxed{4.}$

Abs. Min is $\boxed{0.}$

$f(x) = x^2$ is a polynomial and it is cont. everywhere.

$f(-1) = (-1)^2 = 1$, $f(2) = 2^2 = \boxed{4}$

$f'(x) = 2x \qquad 2x = 0 \qquad x = 0$

C.N.

$f(0) = \boxed{0}$

$(0,0)$

C.P.

Apr 9-9:08 AM

$f(x) = 2x^3 - 54x - 5$, $[0,4]$

Find Abs. max. and abs. min.

$f(x)$ is a Polynomial function $\Rightarrow$ Cont. everywhere

$f(0) = -5$, $f(4) = -93$

C.N.

$f'(x) = 6x^2 - 54$ $\quad f'(x) = 0$ $\quad 6x^2 - 54 = 0$

$x^2 - 9 = 0 \rightarrow x = \pm 3$

-3 is not in $(0,4)$, 3 is in $(0,4)$

$f(3) = -113$ $\quad (3, -113)$ C.P.

Abs. Max is -5

Abs. Min is -113

Apr 9-9:19 AM

$f(x) = 2\cos x + \sin 2x$, $[0, \frac{\pi}{2}]$

$f(x)$ is cont. everywhere

$f(0) = 2\cos 0 + \sin 2(0) = 2 \cdot 1 + 0 = \boxed{2}$

$f(\frac{\pi}{2}) = 2\cos\frac{\pi}{2} + \sin 2(\frac{\pi}{2}) = 2 \cdot 0 + 0 = \boxed{0}$

$f'(x) = -2\sin x + \cos 2x \cdot 2$

$f'(x) = 0$ $\quad -2\sin x + 2\cos 2x = 0$

Recall from Trig. $\quad -\sin x + \cos 2x = 0$

$\cos 2x = \cos^2 x - \sin^2 x$ $\quad -\sin x + 1 - 2\sin^2 x = 0$

$\cos 2x = 2\cos^2 x - 1$ $\quad -2\sin^2 x - \sin x + 1 = 0$

$\boxed{\cos 2x = 1 - 2\sin^2 x}$ $\quad 2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ OR $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\quad \sin x = -1$

C.N. $\rightarrow x = \pi/6$ is in $(0, \frac{\pi}{2})$ $\quad \frac{3\pi}{2}$ Not in $(0, \frac{\pi}{2})$

$f(\frac{\pi}{6}) = 2\cos\frac{\pi}{6} + \sin 2(\frac{\pi}{6})$

$= 2 \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

$= \boxed{\frac{3\sqrt{3}}{2}} \approx 2.598$

Abs. Max $\frac{3\sqrt{3}}{2}$

Abs. Min 0

Apr 9-9:27 AM

Find all critical points for $f(x) = \sqrt[4]{x^3} - 2\sqrt[4]{x}$.

$f'(x)=0$ or undefined

even index, Radicand ≥ 0

$x^3 \geq 0$

$x \geq 0$

Domain $[0, \infty)$

$f(x) = x^{\frac{3}{4}} - 2x^{\frac{1}{4}}$

$f'(x) = \frac{3}{4} \cdot x^{\frac{3}{4}-1} - 2 \cdot \frac{1}{4} x^{\frac{1}{4}-1}$

$= \frac{3}{4} x^{-\frac{1}{4}} - \frac{2}{4} x^{-\frac{3}{4}}$

$f'(x)=0 \qquad \frac{3}{4} x^{-1/4} - \frac{2}{4} x^{-3/4} = 0$

$\frac{1}{4} x^{-3/4} (3x^1 - 2) = 0$ C.N.

$f'(x) = \dfrac{3x - 2}{4\sqrt[4]{x^3}}$

$f'(x) = 0 \rightarrow 3x - 2 = 0 \quad \boxed{x = \frac{2}{3}}$

$f'(x)$ is undefined $\rightarrow 4\sqrt[4]{x^3} = 0 \rightarrow \boxed{x=0}$

Critical Points

$f\left(\frac{2}{3}\right) = \sqrt[4]{\left(\frac{2}{3}\right)^3} - 2\sqrt[4]{\frac{2}{3}} \approx \boxed{} \quad \left(\frac{2}{3}, \right)$ C.P.

$f(0) = \sqrt[4]{0^3} - 2\sqrt[4]{0} = \boxed{0} \quad (0, 0)$

Google Rolle's Theorem and Mean Value Theorem

Apr 9-9:42 AM