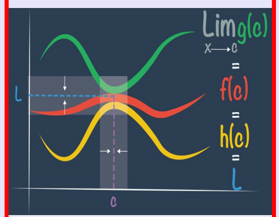
Calculus I

Lecture 3



Feb 19-8:47 AM

Class QZ 1

Given $\boxed{f(x) = x^2 + 2x - 8}$

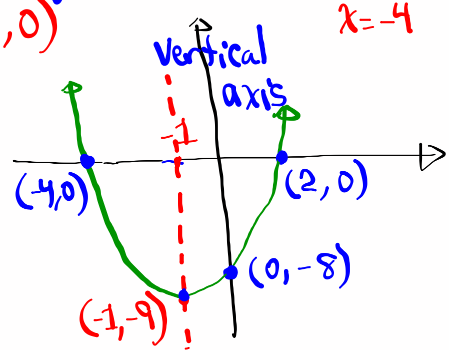
1) Y-Int $\quad f(0) = 0^2 + 2(0) - 8 = -8 \to (0, -8)$ ✓

2) x-Ints $\quad f(x) = 0 \to x^2 + 2x - 8 = 0 \to (x+4)(x-2) = 0$

$x = -4 \quad x = 2$

$\to (-4, 0)$ ✓, $(2, 0)$ ✓

3) Graph.



Feb 6-9:38 AM

Given $x + |y| = 4$

Y-Int. $\to x = 0 \to |y| = 4 \to y = 4,\ y = -4$

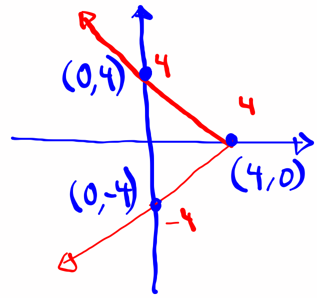
$(0,4), (0,-4)$

X-Int $\to y = 0 \to x = 4$

$(4,0)$

If $y \geq 0 \to |y| = y \to x + y = 4$

If $y < 0 \to |y| = -y \to x - y = 4$



Domain $(-\infty, 4]$

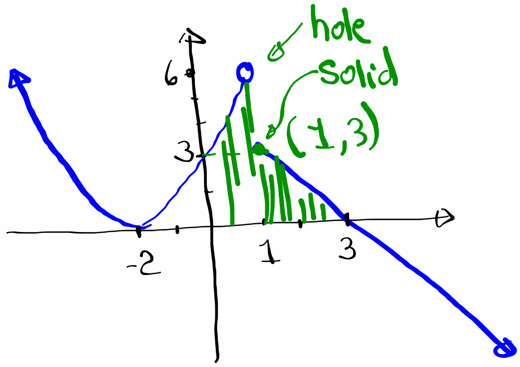
Range $(-\infty, \infty)$

Does this graph belong to a function?

NO, it fails the vertical line test.

Feb 7-8:48 AM

Consider the graph below



1) Does this graph belong to a function?

Yes, by V.L.T.

2) Y-Int $(0,3)$

3) X-Ints

$(-2,0), (3,0)$

4) As $x \to 1^+$, $y \to 3$

5) As $x \to 1^-$, $y \to 6$

6) What is the function value when $x = 1$?

3

Feb 7-8:54 AM

Given $f(x) = |x^2 - 4|$

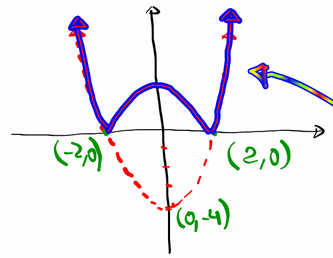
Y-Int $\to x = 0 \to y = |0^2 - 4| = |-4| = 4 \to$ Y-Int $(0,4)$

x-Ints $\to f(x) = 0 \to |x^2 - 4| = 0 \to |(x+2)(x-2)| = 0$

$x = -2$ $\quad x = 2$

Graph

$y = x^2 - 4$



x-Int $(-2,0)$
$(2,0)$

$f(x) = |x^2 - 4|$

office hours after next week

MW 1:00 – 2:30, TTh 4:00 – 6:00

This week and next week MTW F
11:30 – 1:30

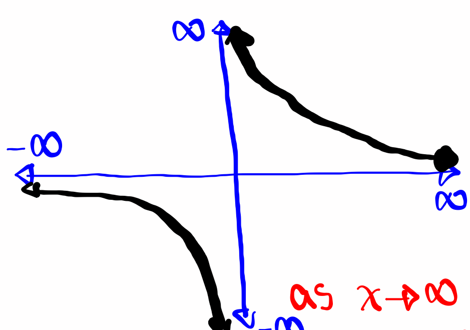
Feb 7-9:00 AM

Given $xy = 1$

Y-Int $\to x = 0 \to 0 \cdot y = 1$ $\quad 0 = 1 \to$ No Y-Int
False

x-Int $\to y = 0 \to x \cdot 0 = 1$ $\quad 0 = 1 \to$ No x-Int

Graph isolate $y \to y = \frac{1}{x}$ $\to x \neq 0$
Domain
$(-\infty, 0) \cup (0, \infty)$



as $x \to 0^+$, $y \to \infty$

as $x \to 0^-$, $y \to -\infty$

as $x \to \infty$, $y \to 0$

as $x \to -\infty$, $y \to 0$

Feb 7-9:11 AM

Distance formula between two Points

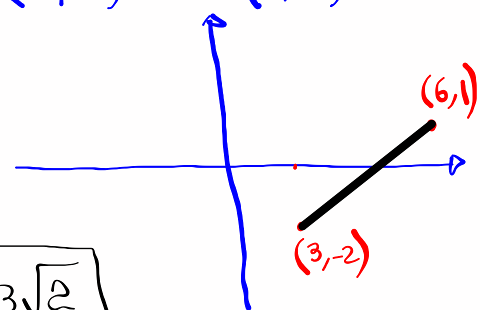
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Find the distance between $(3,-2)$ and $(6,1)$

$$d = \sqrt{(3-6)^2 + (-2-1)^2}$$

$$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$$

$$= \sqrt{9}\sqrt{2} = \boxed{3\sqrt{2}}$$

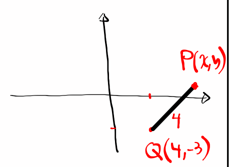


Feb 7-9:17 AM

Given $Q(4,-3)$ and $P(x,y)$,

$d(P,Q) = 4$

Find the equation for all points (x,y) that meets these Conditions.



$$\underbrace{d(P,Q)}_{4} = \sqrt{(x-4)^2 + (y - -3)^2}$$

$$4 = \sqrt{(x-4)^2 + (y+3)^2}$$

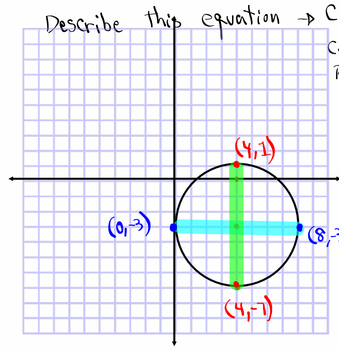
Let's square both sides

$$16 = (x-4)^2 + (y+3)^2$$

Describe this equation → Circle

Center $(4,-3)$

Radius 4



Domain $[0,8]$

Range $[-7,1]$

Feb 7-9:21 AM

$f(x) = \frac{1}{x}$ Difference Quotient

Find $\frac{f(x+h) - f(x)}{h}$, Simplify, then evaluate for $h = 0$.

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{x\cancel{(x+h)} \cdot \frac{1}{\cancel{x+h}} - \cancel{x}(x+h) \cdot \frac{1}{\cancel{x}}}{x(x+h) \cdot h}$$

LCD $= x(x+h)$

$$= \frac{x - (x+h)}{x(x+h)h} = \frac{\cancel{x} - \cancel{x} - h}{x(x+h)h}$$

$$= \frac{-\cancel{h}}{x(x+h)\cancel{h}} = \frac{-1}{x(x+h)}$$

when $h = 0 \rightarrow \frac{-1}{x(x+0)} = \boxed{\frac{-1}{x^2}}$

Feb 7-9:31 AM

Given $f(x) = x^2$

Find the difference quotient, Simplify, evaluate for $h=0$.

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x + h$$

For $h = 0 \rightarrow \boxed{2x}$

Feb 7-9:37 AM

$f(x) = \sqrt{x}$

Evaluate $\underbrace{\text{the difference quotient}}$ for $h=0$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

For $h=0$

$\frac{0}{0}$

indeterminate form

$$= \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

Recall from Algebra

$(A-B)(A+B) = A^2 - B^2$

$$= \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{\cancel{x} + \cancel{h}^{1} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

For $h=0$ $\Rightarrow \frac{1}{\sqrt{x+0} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$

SG 0 is due Thursday

Your Submission must be one file, portrait style, and Pages in order.

If not, will not be graded.

Work on SG 1