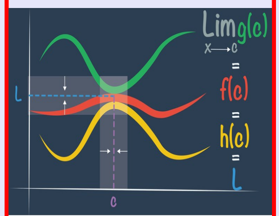
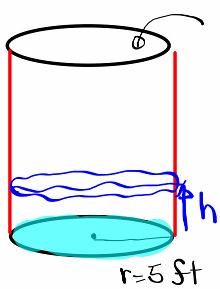
Calculus I

Lecture 29



Feb 19-8:47 AM

A cylindrical tank has a radius of 5 ft and it is being filled at the rate of 3 ft^3/min.



$$\frac{dV}{dt} = 3 \text{ ft}^3/\text{min.}$$

$$\frac{dh}{dt} = +$$

$$V = \pi r^2 h$$

$$V = \pi \cdot 5^2 h$$

$$V = 25\pi h$$

$$\frac{dV}{dt} = \frac{d}{dt}\left[25\pi h\right]$$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$$

$$3 = 25\pi \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{3}{25\pi} \text{ ft/min}}$$

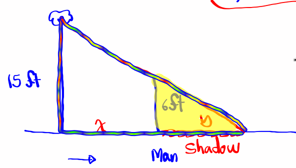
height is increasing at rate of $\frac{3}{25\pi}$ ft/min.

Mar 27-8:49 AM

A Street light is 15-ft-tall.

A man who is 6-ft-tall walks away from the light at rate of 5 ft/sec.

How fast is the $\underbrace{\text{length of his shadow}}_{y}$ changing?



$\frac{dx}{dt} = 5 \text{ ft/s}$, $\frac{dy}{dt} = ?$

$$\frac{y}{6} = \frac{x+y}{15}$$

$$15y = 6(x+y)$$

$$15y = 6x + 6y$$

$$9y = 6x$$

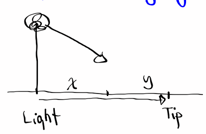
$$3y = 2x$$

$$3\frac{dy}{dt} = 2\frac{dx}{dt}$$

$$3\frac{dy}{dt} = 2 \cdot 5$$

$$\frac{dy}{dt} = \frac{10}{3} \text{ ft/sec.}$$

what if the question was how fast $\boxed{\text{the tip}}$ of his shadow changing?



x is changing

y is changing

$$\frac{d}{dt}[x+y] =$$

$$\frac{dx}{dt} + \frac{dy}{dt} = 5 + \frac{10}{3}$$

$$= \frac{25}{3} \text{ ft/Sec.}$$

Mar 27-8:57 AM

$$f(x) = \frac{2x^2}{x^2-1}$$

Please Submit this as class QZ 16

1) Find $f'(x)$.

2) Find x-values where $f'(x) = 0$ or undefined.

3) Find $f''(x)$

2) Find x-values where $f''(x) = 0$ or undefined.

Make sure to do this neatly in one page, ready to submit when class starts tomorrow.

Mar 26-9:50 AM

$$f(x) = \frac{2x^2}{x^2-1}$$

$$f(0) = \frac{2(0)^2}{0^2-1} = 0$$

x-Int $(0,0)$

Y-Int $(0,0)$

$$f(-x) = \frac{2(-x)^2}{(-x)^2-1} = \frac{2x^2}{x^2-1} = f(x)$$

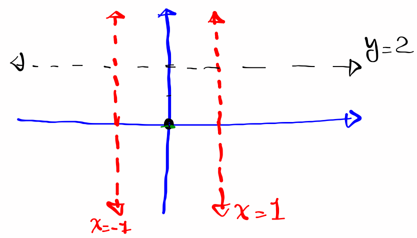
Since $f(-x) = f(x)$, $f(x)$ is an even funct.

Sym. w/t Y-axis.

Domain $x^2-1 \neq 0$ $x^2 \neq 1$ $x \neq \pm 1$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x^2}{x^2-1} = \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \lim_{x\to\infty} \frac{2}{1-\cancel{\frac{1}{x^2}}^{\,0}} = \boxed{2}$$

$$\lim_{x\to-\infty} f(x) = \text{- - - -} = \boxed{2}$$



Mar 27-9:11 AM

$$f(x) = \frac{2x^2}{x^2-1} = \frac{2x^2-2+2}{x^2-1} = \frac{2x^2-2}{x^2-1} + \frac{2}{x^2-1}$$

Find $f'(x)$

$$= \frac{2\cancel{(x^2-1)}}{\cancel{x^2-1}} + \frac{2}{x^2-1}$$

$$f(x) = 2 + \frac{2}{x^2-1}$$

$$f(x) = 2 + 2(x^2-1)^{-1}$$

$$f'(x) = 0 + 2\cdot -1(x^2-1)^{-1-1} \cdot 2x = -4x(x^2-1)^{-2} = \frac{-4x}{(x^2-1)^2}$$

$$f'(x) = \frac{-4x}{(x^2-1)^2}$$

$f'(x) = 0$ $\frac{-4x}{(x^2-1)^2} = 0 \rightarrow -4x = 0$ $\boxed{x=0}$

$f'(x)$ is undefined

$(x^2-1)^2 = 0 \rightarrow x^2-1=0$

$x = \pm 1$

Mar 27-9:20 AM

$$f'(x) = \frac{-4x}{(x^2-1)^2}$$

$$f''(x) = \frac{\frac{d}{dx}[-4x]\cdot(x^2-1)^2 - -4x\cdot\frac{d}{dx}\left[(x^2-1)^2\right]}{\left[(x^2-1)^2\right]^2}$$

$$= \frac{-4(x^2-1)^2 + 4x\cdot 2\cdot(x^2-1)^1\cdot 2x}{(x^2-1)^4}$$

$$= \frac{\cancel{(x^2-1)}\left[-4(x^2-1)+16x^2\right]}{(x^2-1)^{\cancel{4}\,3}} = \frac{-4x^2+4+16x^2}{(x^2-1)^3}$$

$$\boxed{f''(x) = \frac{12x^2+4}{(x^2-1)^3}}$$

$f''(x)=0 \to 12x^2+4=0$

No real Solution

$12x^2+4>0$

$f''(x)$ is undefined where

$(x^2-1)^3=0 \to x=\pm 1$

Mar 27-9:27 AM

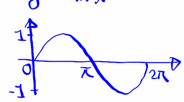
A particle is moving along the curve $y=2\sin\frac{\pi x}{2}$

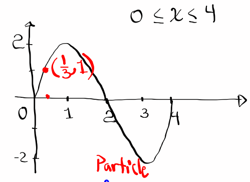
As it passes the point $\left(\frac{1}{3},1\right)$, the x-Coordinate increases at the rate of $\sqrt{10}$ cm/s. $\frac{dx}{dt}=\sqrt{10}$ cm/s

How fast is the distance from the particle to the origin changing at that instant? $\frac{dz}{dt}=?$

$y=\sin x$ $\qquad 0\le\frac{\pi x}{2}\le 2\pi \qquad 0\le\pi x\le 4\pi$

$0\le x\le 4$





Verify $\left(\frac{1}{3},1\right)$ being on that graph

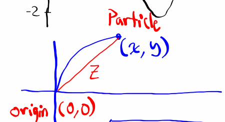
$y=2\sin\frac{\pi x}{2}$

$1=2\sin\frac{\pi}{2}\cdot\frac{1}{3}$

$1=2\sin\frac{\pi}{6}$

$1=2\sin 30^\circ$

$1=\cancel{2}\cdot\frac{1}{\cancel{2}}$ ✓



$z=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$z=\sqrt{(x-0)^2+(y-0)^2}$

$z=\sqrt{x^2+y^2}$

$z=\sqrt{x^2+\left(2\sin\frac{\pi x}{2}\right)^2}$

$z=\sqrt{x^2+4\sin^2\frac{\pi x}{2}}$

$z^2=x^2+4\sin^2\frac{\pi x}{2}$

$\frac{d}{dt}\left[z^2\right]=\frac{d}{dt}\left[x^2+4\sin^2\frac{\pi x}{2}\right]$

Mar 27-9:36 AM

$\frac{d}{dt}[z^2] = \frac{d}{dt}\left[x^2 + 4\sin^2\frac{\pi x}{2}\right]$

$\frac{d}{dt}\left[\left(\sin\frac{\pi x}{2}\right)^2\right]$

$2z\,\frac{dz}{dt} = 2x \cdot \frac{dx}{dt} + \cancel{4}^{2} \cdot 2 \cdot \sin\frac{\pi x}{2} \cdot \cos\frac{\pi x}{2} \cdot \frac{\pi}{\cancel{2}} \cdot \frac{dx}{dt}$

Find $\frac{dz}{dt}$ at $\left(\frac{1}{3}, 1\right)$, given $\frac{dx}{dt} = \sqrt{10}$

$z\,\frac{dz}{dt} = \frac{1}{3} \cdot \sqrt{10} + 2 \cdot \sin\frac{\pi}{6} \cdot \cos\frac{\pi}{6} \cdot \sqrt{10}$

Find z

$z = \sqrt{x^2 + 4\sin^2\frac{\pi x}{2}}$

$x = \frac{1}{2}$, find z.