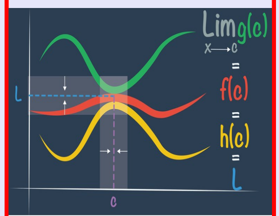
Calculus I

Lecture 28



Feb 19-8:47 AM

Class QZ 15

Suppose $y = \sqrt{2x+1}$ and $\frac{dx}{dt} = 3$

Find $\frac{dy}{dt}$ when $x = 4$.

Method I:

$y = \sqrt{2x+1}$ $\qquad y = (2x+1)^{\frac{1}{2}}$

$$\frac{dy}{dt} = \frac{1}{\cancel{2}}(2x+1)^{-1/2} \cdot \cancel{2} \cdot \frac{dx}{dt}$$

$$= \frac{1}{\sqrt{2x+1}} \cdot \frac{dx}{dt} = \frac{1}{\sqrt{2(4)+1}} \cdot 3$$

$$= \frac{1}{3} \cdot 3 = \boxed{1}$$

Method II:

$y = \sqrt{2x+1} \rightarrow y^2 = 2x+1$

$$2y\frac{dy}{dt} = 2\frac{dx}{dt}$$

$x = 4$

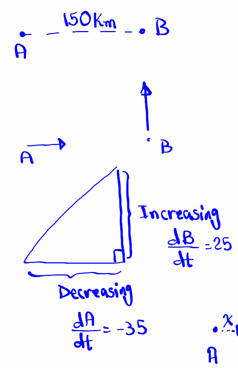
$y = \sqrt{2 \cdot 4 + 1}$

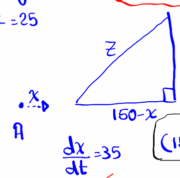
$= 3$

$$\frac{dy}{dt} = \frac{1}{y} \cdot \frac{dx}{dt} = \frac{1}{3} \cdot 3 = \boxed{1}$$

Mar 26-8:23 AM

At noon, ship A is 150 Km west of ship B. ship A is sailing East at 35 Km/hr. ship B is Sailing north at 25 Km/hr. How fast is the distance between them changing at 4:00 PM?





$$\boxed{(150-x)^2+y^2=z^2}$$

$$2(150-x)^1\cdot-\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

at 4:00 PM

$x=4(35)=140$

$y=4(25)=100$

$(150-140)^2+100^2=z^2$

$z=\sqrt{10,100}$

$$(150-140)\cdot{-35}+100\cdot25=\sqrt{10100}\,\frac{dz}{dt}$$

$$\frac{dz}{dt}=\frac{2150}{\sqrt{10100}}$$

≈ 21.4 Km/hr.

Mar 26-9:04 AM

The height of a triangle is increasing at 1 cm/min. $\frac{dh}{dt}=1$ $\frac{dA}{dt}=2$

The area is increasing at 2 cm²/min.

At what rate is the base changing when h=10cm and A=100cm²?

$A=\frac{bh}{2}$ $\qquad 2A=bh$

$100=\frac{b\cdot10}{2}$ $\qquad 2\cdot\frac{dA}{dt}=\frac{db}{dt}\cdot h+b\cdot\frac{dh}{dt}$

$\boxed{b=20}$ $\qquad 2\cdot2=\frac{db}{dt}\cdot10+20\cdot1$

$$4-20=10\frac{db}{dt}$$

base is decreasing. $\boxed{\frac{db}{dt}=-1.6\text{ cm/min}}$

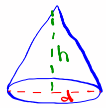
Mar 26-9:19 AM

Gravel is being dumped at rate of $30 ft^3/min$

$\frac{dV}{dt} = 30\ ft^3/min.$

It forms a pile in shape of a coin such that height is equal to its diameter.

How fast is the height changing when the pile is $\boxed{10\ ft\ high}$?



$d = h$

Volume of a right-Circular Cone

$d = 2r$

$h = 2r$

$r = \frac{h}{2}$

$V = \frac{\pi r^2 h}{3}$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi \cdot \frac{h^3}{4}$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{\cancel{12}}\pi \cdot \cancel{3}h^2 \cdot \frac{dh}{dt}$

$30 = \frac{1}{4} \cdot \pi \cdot 10^2 \frac{dh}{dt}$

$120 = 100\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{\cancel{12}^{6}}{\cancel{10}_{5}\pi}$

$\boxed{\frac{dh}{dt} = \frac{6}{5\pi}\ ft/min.}$

Mar 26-9:25 AM

$f(x) = \sqrt[3]{x^3+1} \longrightarrow f(x) = (x^3+1)^{1/3}$

$f(0) = \sqrt[3]{0^3+1} = \sqrt[3]{1} = 1$

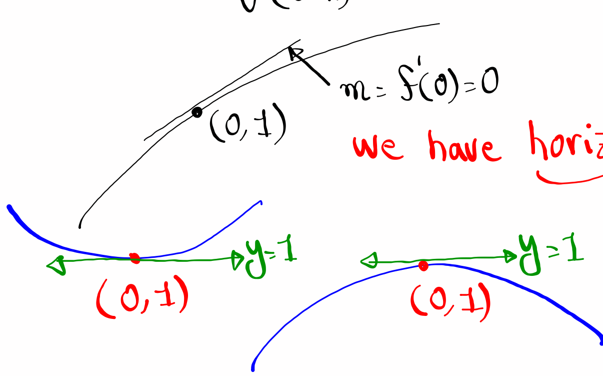
$f'(x) = \frac{1}{\cancel{3}}(x^3+1)^{1/3 - 1} \cdot \cancel{3}x^2$

Find $f'(0)$

$f'(x) = \frac{x^2}{(x^3+1)^{2/3}}$

$= \frac{x^2}{\sqrt[3]{(x^3+1)^2}}$

$f'(0) = \frac{0^2}{\sqrt[3]{(0^3+1)^2}} = 0$



We have $\underbrace{\text{horizontal tan. line}}_{m=0}$

Mar 26-9:35 AM

$$f(x) = \left(\frac{\cos x}{2+\sin x}\right)^2$$

$$f(0) = \left(\frac{\cos 0}{2+\sin 0}\right)^2 = \left(\frac{1}{2+0}\right)^2 = \frac{1}{4}$$

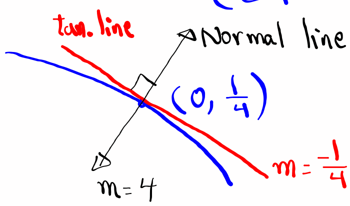
$$f'(x) = 2\left(\frac{\cos x}{2+\sin x}\right)^1 \cdot \frac{-\sin x\cdot(2+\sin x) - \cos x \cdot \cos x}{(2+\sin x)^2}$$

$$= \frac{2\cos x \cdot \left[-2\sin x - \sin^2 x - \cos^2 x\right]}{(2+\sin x)^3}$$

$$= \frac{2\cos x\left[-2\sin x - 1\right]}{(2+\sin x)^3}$$

$$f'(0) = \frac{2\cdot 1\left[-2\cdot 0 - 1\right]}{(2+0)^3}$$

$$f'(0) = \frac{-2}{8} = \boxed{\frac{-1}{4}}$$



$$y - \frac{1}{4} = 4(x-0)$$

$$\boxed{y = 4x + \frac{1}{4}}$$

Mar 26-9:41 AM

$$f(x) = \frac{2x^2}{x^2-1}$$

1) Find $f'(x)$.

2) Find x-values where $f'(x)=0$ or undefined.

3) Find $f''(x)$

2) Find x-values where $f''(x)=0$ or undefined.

Make sure to do this neatly in one page, ready to submit when class starts tomorrow.

Mar 26-9:50 AM