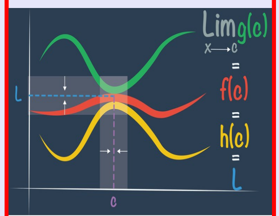
Calculus I

Lecture 27



Feb 19-8:47 AM

Given $x^2 + y^2 + z^2 = 9$,

$\frac{dx}{dt} = 5$, $\frac{dy}{dt} = -4$

Find $\frac{dz}{dt}$ when $(x, y, z) = (2, 1, 2)$

$$\frac{d}{dt}\left[x^2 + y^2 + z^2\right] = \frac{d}{dt}[9]$$

$$\frac{d}{dt}[x^2] + \frac{d}{dt}[y^2] + \frac{d}{dt}[z^2] = 0$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} + 2z \cdot \frac{dz}{dt} = 0$$

$$2 \cdot 5 + 1 \cdot (-4) + 2 \cdot \frac{dz}{dt} = 0$$

$$10 - 4 + 2\frac{dz}{dt} = 0 \qquad \boxed{\frac{dz}{dt} = -3}$$

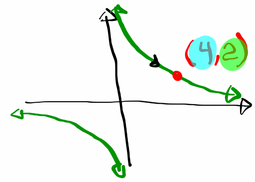
It is decreasing

Mar 25-8:47 AM

A particle is moving along the curve given by $xy = 8$ $\longrightarrow y = \frac{8}{x}$

when it reaches $(4,2)$, y-coordinate is decreasing at the rate of 3 cm/s. $\frac{dy}{dt} = -3$

How fast is $\boxed{x\text{-coordinate}}$ changing at that instant? $\frac{dx}{dt} = ?\ +$



$xy = 8$

$\frac{d}{dt}[xy] = \frac{d}{dt}[8]$

Product Rule

$\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = 0$

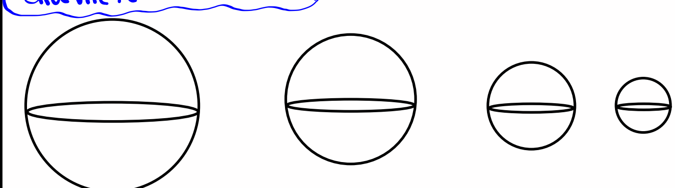
$\frac{dx}{dt} \cdot 2 + 4 \cdot (-3) = 0$

$\boxed{\frac{dx}{dt} = 6 \text{ cm/s.}}$ It is increasing.

Mar 25-8:52 AM

If a snowball melts so its surface area decreases at $1 \text{ cm}^2/\text{min}$. $\frac{dA}{dt} = -1 \text{ cm}^2/\text{min}$ $A = 4\pi r^2$

How fast is its diameter changing when diameter is 10 cm? $\longrightarrow d = 2r \rightarrow r = \frac{d}{2}$



Sphere $V = \frac{4\pi r^3}{3}$ $A = 4\pi r^2$

$A = 4\pi\left(\frac{d}{2}\right)^2$ $A = \cancel{4}\pi \cdot \frac{d^2}{\cancel{4}}$ $A = \pi d^2$

diameter

$\frac{dA}{dt} = \pi \cdot 2d \cdot \frac{dd}{dt}$

$-1 = 2\pi \cdot 10 \cdot \frac{d}{dt}[d]$

$\frac{d}{dt}[d] = \frac{-1}{20\pi}$ cm/min.

It is decreasing.

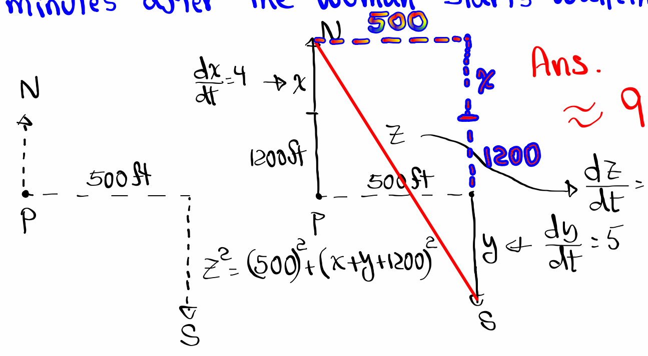
Mar 25-9:04 AM

A man starts walking north at 4 ft/s from a point P.

$5 \cdot 60 \cdot 4 = 1200$ ft

5 minutes later, a woman starts walking South at 5 ft/s from a point 500 ft east of P.

At what rate the people moving apart 15 minutes after the woman starts walking?



$z^2 = (500)^2 + (x + y + 1200)^2$

$\frac{dz}{dt} = ?$

in 15 minutes time for woman

Ans. ≈ 9 ft/s.

Mar 25-9:11 AM

$f(x) = x^4 - 4x^3$

1) Find $f'(x)$, and Solve $f'(x) = 0$

$f'(x) = 4x^3 - 12x^2$

$4x^3 - 12x^2 = 0$

$4x^2(x - 3) = 0$

$x^2 = 0 \rightarrow x = 0$

$x - 3 = 0 \rightarrow x = 3$

2) Find $f''(x)$, and Solve $f''(x) = 0$

$f''(x) = 12x^2 - 24x$

$12x^2 - 24x = 0$

$12x(x - 2) = 0$

$x = 0 \rightarrow x = 0$

$x - 2 = 0 \rightarrow x = 2$

3) Sign chart

x	$-\infty$ to 0	0 to 2	2 to 3	3 to ∞
$f'(x)$	−	−	−	+
$f''(x)$	+	−	+	+

Mar 25-9:24 AM

$f(x) = \dfrac{x^3}{x^2+1}$

1) Find $f'(x)$

$$f'(x) = \frac{3x^2(x^2+1) - x^3 \cdot 2x}{(x^2+1)^2} = \frac{\overbrace{3x^4+3x^2-2x^4}^{x^4+3x^2}}{(x^2+1)^2} = \frac{x^2(x^2+3)}{(x^2+1)^2}$$

2) Discuss all x-values where $f'(x)=0$ or undefined.

$$f'(x)=0 \rightarrow \frac{x^2(x^2+3)}{(x^2+1)^2}=0 \rightarrow x^2(\underbrace{x^2+3}_{\text{Never Zero}})=0 \rightarrow x^2=0 \quad \boxed{x=0}$$

$f'(x)$ is undefined when $(\underbrace{x^2+1}_{\text{Never Zero.}})^2=0$

3) Find $f''(x)$

$$f'(x) = \frac{x^4+3x^2}{(x^2+1)^2} \qquad f''(x) = \frac{(4x^3+6x)\cdot(x^2+1)^2 - (x^4+3x^2)\cdot 2(x^2+1)\cdot 2x}{[(x^2+1)^2]^2}$$

$$f''(x) = \frac{\cancel{(x^2+1)}\cdot[(4x^3+6x)\cdot(x^2+1) - (x^4+3x^2)\cdot 4x]}{(x^2+1)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{2x(2x^2+3)\cdot(x^2+1) - \overset{2x^2}{4x^3}(x^2+3)}{(x^2+1)^3}$$

$$= \frac{2x[\cancel{2x^4} + 2x^2 + 3x^2 + 3 - \cancel{2x^4} - 6x^2]}{(x^2+1)^3}$$

$$\boxed{f''(x) = \frac{2x(3-x^2)}{(x^2+1)^3}}$$

$f''(x)=0 \rightarrow x=0,$
$x = \pm\sqrt{3}$

$f''(x)$ undefined
$(x^2+1)^3=0 \rightarrow$ Never

Mar 25-9:34 AM

Sign chart

x	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		$+\infty$
$f'(x)$		$+$		$+$		$+$		$+$	
$f''(x)$		$+$		$-$		$+$		$-$	

$$f'(x) = \frac{x^2(x^2+3)}{(x^2+1)^2} \qquad f''(x) = \frac{2x(3-x^2)}{(x^2+1)^3}$$

Google Concavity on the graph of a function

Mar 25-9:50 AM