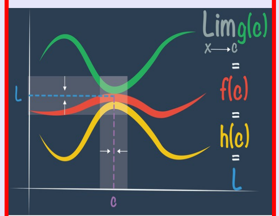
Calculus I

Lecture 26



Feb 19-8:47 AM

Class QZ 14

Box Your final Ans

Use linear approximation to estimate $(5.01)^3$.

$$\boxed{f(x) \approx f(a) + f'(a)(x-a)}$$

$f(x) = x^3$

$a = 5$

$f'(x) = 3x^2$

$f(a) = 5^3 = 125$

$f'(a) = 3 \cdot 5^2 = 75$

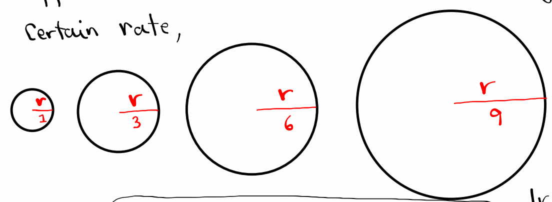
$x^3 \approx 125 + 75(x-5)$

$5.01^3 \approx 125 + 75(5.01 - 5)$

$= 125 + 75(.01) \approx \boxed{125.75}$

Mar 20-9:48 AM

Suppose the radius of a circle is increasing at certain rate,



Suppose radius increasing at 3 cm/min. $\rightarrow \frac{dr}{dt} = 3$ cm/min

How fast is the area changing when $r = 5$ cm?

$A = \pi r^2$

Take derivative of both sides with respect to time.

$\frac{dA}{dt} = \frac{d}{dt}[\pi r^2]$

$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$

$= 2\pi(5) \cdot 3$

$\boxed{\frac{dA}{dt} = 30\pi \ \text{cm}^2/\text{min.}}$

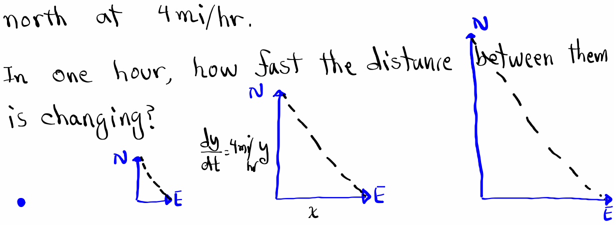
Related Rates

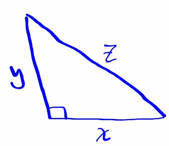
Mar 21-9:04 AM

Two people leave an intersection.

one going east at 3 mi/hr, other one is going north at 4 mi/hr.

In one hour, how fast the distance between them is changing?





$x^2 + y^2 = z^2$

take derivative with respect to time from both sides

$\frac{dx}{dt} = 3$ mi/hr

$\frac{dy}{dt} = 4$ mi/hr

In 1 hr, $x = 3$, $y = 4$

$3^2 + 4^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\boxed{z = 5}$

$3 \cdot 3 + 4 \cdot 4 = 5\frac{dz}{dt}$

$9 + 16 = 5\frac{dz}{dt}$

$\frac{25}{5} = \frac{dz}{dt} \rightarrow 5$ mi/hr

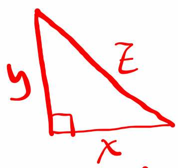
Mar 21-9:11 AM

Given $x^2 + y^2 = z^2$

$\frac{dx}{dt} = 6$

$\frac{dy}{dt} = -8$

$x = 3$, $y = 4$



1) Find $z > 0$ $\to$ $3^2 + 4^2 = z^2$ $\boxed{z = 5}$

2) Find $\frac{dz}{dt}$ when $x = 3$ & $y = 4$.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$3 \cdot 6 + 4 \cdot (-8) = 5\frac{dz}{dt}$$

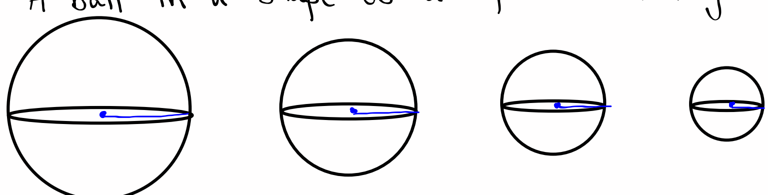
$$18 - 32 = 5\frac{dz}{dt}$$

$$-14 = 5\frac{dz}{dt} \to \frac{dz}{dt} = -2.8$$

Since $\frac{dz}{dt} < 0$, z is decreasing

Mar 21-9:21 AM

A ball in a shape of a sphere is leaking air.



Its radius is decreasing at the rate of 2 cm/min. $\frac{dr}{dt} = -2$ cm/min

At what rate its volume is decreasing $\to$ when $r = 5$ cm?

Volume of Sphere

$$V = \frac{4\pi r^3}{3}$$

take derivative with respect to time, then evaluate for $r = 5$.

$$\frac{dV}{dt} = \frac{d}{dt}\left[\frac{4\pi r^3}{3}\right]$$

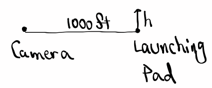
$$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$$

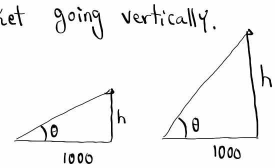
$$\to \frac{dV}{dt} = 4\pi(5)^2 \cdot (-2)$$

$$= -200\pi \text{ cm}^3/\text{min}.$$

Mar 21-9:27 AM

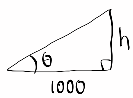
A Camera is located 1000 ft from the Launching Pad of rocket going vertically.





height of the rocket is increasing at 200 ft/min

How fast is the angle changing in 5 minutes?



$\tan\theta = \frac{h}{1000}$ $\qquad \frac{dh}{dt} = 200$ ft/min.

Take derivative of both sides with respect to time.

In 5 minutes $h = 1000$

when $h = 1000$

$\tan\theta = 1$

$\theta = 45°$

$\sec 45° = \sqrt{2}$

$\sec^2 45° = 2$

$$\tan\theta = \frac{h}{1000}$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{1000} \cdot \frac{dh}{dt}$$

$$\sec^2 45° \cdot \frac{d\theta}{dt} = \frac{1}{1000} \cdot 200$$

$$2 \cdot \frac{d\theta}{dt} = \frac{1}{5} \qquad \frac{d\theta}{dt} = \frac{1}{10} \text{ Rad/min}$$

Mar 21-9:38 AM

$$A = \frac{bh}{2} \qquad \frac{db}{dt} = 4, \quad \frac{dh}{dt} = -5$$

Find $\frac{dA}{dt}$ when $b = 5$ and $h = 4$.

$$\frac{dA}{dt} = \frac{d}{dt}\left[\frac{bh}{2}\right]$$

$$= \frac{1}{2}\frac{d}{dt}[bh] = \frac{1}{2}\left[\frac{db}{dt} \cdot h + b \cdot \frac{dh}{dt}\right]$$

$$= \frac{1}{2}[4 \cdot 4 + 5 \cdot (-5)]$$

$$= \frac{1}{2}[16 - 25] = \frac{1}{2}(-9) = \frac{-9}{2}$$

$$= \boxed{-4.5}$$

A is decreasing.

Mar 21-9:49 AM