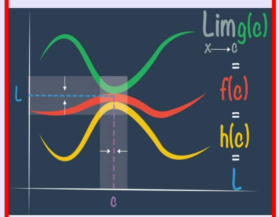
Calculus I

Lecture 24

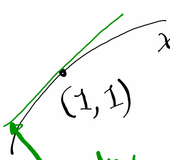


Feb 19-8:47 AM

Class QZ 13

Given $\boxed{x^2 + xy + y^2 = 3}$

Find $\left.\frac{dy}{dx}\right|_{(1,1)}$



$(1,1)$

$x^2+xy+y^2=3$

$1^2+1\cdot1+1^2=3$

$3=3$

$m = \left.\frac{dy}{dx}\right|_{(1,1)} = -1$

$y-1 = -1(x-1)$

$y = -x+1+1$

$\boxed{y = -x+2}$

$\frac{d}{dx}\left[x^2+xy+y^2\right] = \frac{d}{dx}[3]$

$\frac{d}{dx}\left[x^2\right] + \frac{d}{dx}[xy] + \frac{d}{dx}\left[y^2\right] = 0$

$2x + 1\cdot y + x\cdot\frac{dy}{dx} + 2y\cdot\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$

$(x+2y)\boxed{\frac{dy}{dx}} = -2x-y$

$$\boxed{\frac{dy}{dx} = \frac{-2x-y}{x+2y}}$$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2\cdot1-1}{1+2\cdot1} = \frac{-3}{3}$

$= \boxed{-1}$ ✓

Mar 14-9:45 AM

Given $y\cos x = x^2 + y^2$

Find $\frac{dy}{dx}$.

$$\frac{d}{dx}[y\cos x] = \frac{d}{dx}[x^2+y^2]$$

Product Rule

Sum

$$\frac{d}{dx}[y]\cdot\cos x + y\cdot\frac{d}{dx}[\cos x] = \frac{d}{dx}[x^2] + \frac{d}{dx}[y^2]$$

$$\frac{dy}{dx}\cdot\cos x - y\cdot\sin x = 2x + 2y\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx}\cdot\cos x - 2y\cdot\frac{dy}{dx} = 2x + y\cdot\sin x$$

$$(-2y+\cos x)\frac{dy}{dx} = 2x + y\sin x$$

$$\boxed{\frac{dy}{dx} = \frac{2x+y\sin x}{-2y+\cos x}}$$

Mar 19-8:46 AM

Given $\boxed{x\sin y} + \boxed{y\sin x} = \boxed{1}$

Find $\frac{dy}{dx}$

$$\boxed{1\cdot\sin y + x\cdot\cos y\cdot\frac{dy}{dx}} + \boxed{\frac{dy}{dx}\cdot\sin x + y\cdot\cos x} = \boxed{0}$$

$$\sin y + x\cos y\cdot\frac{dy}{dx} + \sin x\cdot\frac{dy}{dx} + y\cos x = 0$$

$$(x\cos y + \sin x)\frac{dy}{dx} = -\sin y - y\cos x$$

$$\boxed{\frac{dy}{dx} = \frac{-\sin y - y\cos x}{x\cos y + \sin x}} = -\frac{\sin y + y\cos x}{x\cos y + \sin x}$$

Mar 19-8:52 AM

Given $\sin(x+y) = 2x - 2y$

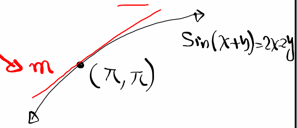
1) Is (π,π) on the graph of the given eqn?

$\sin(\pi+\pi) = 2\pi - 2\pi \rightarrow 0 = 0 \checkmark$

$\sin 2\pi = 0$

$\underline{\underline{\text{Yes}}}$

2) Find $\left.\dfrac{dy}{dx}\right|_{(\pi,\pi)} \rightarrow m$



$\sin(x+y) = 2x - 2y$

$\cos(x+y)\cdot\left[1 + \dfrac{dy}{dx}\right] = 2 - 2\cdot\dfrac{dy}{dx}$

Plug in (π,π)

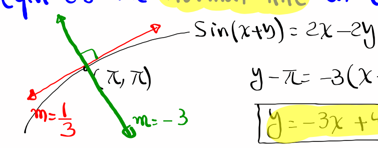
$m = \left.\dfrac{dy}{dx}\right|_{(\pi,\pi)}$

$\overset{1}{\cancel{\cos 2\pi}}\cdot[1 + m] = 2 - 2m$

$1 + m = 2 - 2m$

$3m = 1$ $\quad \boxed{m = \frac{1}{3}}$

Find eqn of the normal line at (π,π)

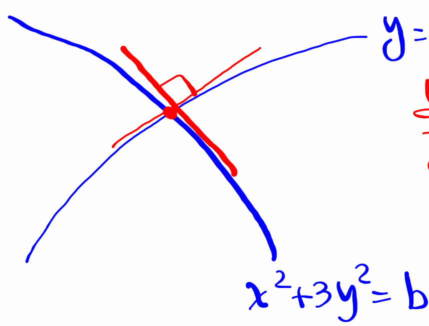


$y - \pi = -3(x - \pi)$

$\boxed{y = -3x + 4\pi}$

Mar 19-9:01 AM

Show $y = ax^3$ and $x^2 + 3y^2 = b$ are orthogonal curves at their intersection points.



$y = ax^3$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left[ax^3\right] = \boxed{3ax^2}$

$\dfrac{d}{dx}\left[x^2+3y^2\right] = \dfrac{d}{dx}[b]$

$2x + 6y\cdot\dfrac{dy}{dx} = 0$

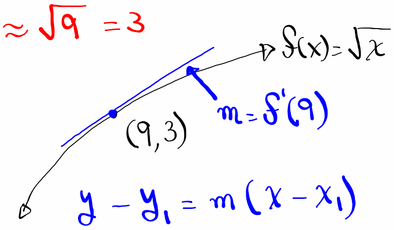
$\dfrac{dy}{dx} = \dfrac{-2x}{6y} = \boxed{\dfrac{-x}{3y}}$

$\cancel{3}ax^2 \cdot \dfrac{-x}{\cancel{3}y} = \dfrac{-ax^3}{y} = \dfrac{-y}{y} = \boxed{-1}$

Mar 19-9:09 AM

Introductions to linear approximation

Estimate $\sqrt{9.1} \approx \sqrt{9} = 3$



$f(x)=\sqrt{x}$ $\quad f(9)=3$

$f(9.1)=\sqrt{9.1}$

$f'(x)=\frac{1}{2\sqrt{x}}$

$f'(9)=\frac{1}{2\sqrt{9}}=\frac{1}{6}$

Use calc to find

$\sqrt{9.1}\approx 3.017$

$y-y_1=m(x-x_1)$

$\sqrt{x}-f(9)\approx f'(9)(x-9)$

$\sqrt{x}\approx f(9)+f'(9)(x-9)$

$\sqrt{x}\approx 3+\frac{1}{6}(x-9)$

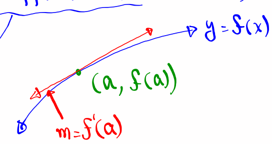
$\sqrt{9.1}\approx 3+\frac{1}{6}(9.1-9)$

$=3+\frac{1}{6}(.1)=3+\frac{1}{60}$

$=\frac{181}{60}\approx \boxed{3.017}$

Mar 19-9:17 AM

Linear Approximation of $f(x)$ near $x=a$:



$f(x)\approx f(a)+f'(a)(x-a)$

Use linear approximation to estimate $\sqrt[3]{1001}$.

$f(x)=\sqrt[3]{x}$

$a=1000$

$\sqrt[3]{1000}=10$

$\sqrt[3]{1001}\approx\sqrt[3]{1000}=10$

$f(x)=\sqrt[3]{x}$ $\quad f(x)=x^{1/3}$

$f'(x)=\frac{1}{3}x^{\frac{1}{3}-1}$

$=\frac{1}{3}x^{-2/3}$

$f'(x)=\frac{1}{3\sqrt[3]{x^2}}$

Near $a=1000$ ✓

$f(a)=\sqrt[3]{1000}=10$ ✓

$f'(a)=\frac{1}{3\sqrt[3]{1000^2}}=\frac{1}{3\cdot 10^2}=\frac{1}{300}$ ✓

$f(x)\approx f(a)+f'(a)(x-a)$

$\boxed{\sqrt[3]{x}\approx 10+\frac{1}{300}(x-1000)}$

$\sqrt[3]{x}\approx 10+\frac{1}{300}(x-1000)$

$\sqrt[3]{1001}\approx 10+\frac{1}{300}(1001-1000)$

$=10+\frac{1}{300}\cdot 1$

Now use your calc to find

$\sqrt[3]{1001}\approx 10.003$ $\quad =10+\frac{1}{300}=\frac{3001}{300}\approx\boxed{10.003}$

Mar 19-9:25 AM

Estimate $\tan 46^\circ$ to 3-decimal places.

$\tan 46^\circ \approx \tan 45^\circ = 1$

Use your calc to find $\boxed{\tan 46^\circ \approx 1.036}$

$f(x) = \tan x$

Near $a = 45^\circ = \frac{\pi}{4}$

$f(a) = \tan 45^\circ = 1$

$f'(x) = \sec^2 x$

$f'(a) = \sec^2 45^\circ = \frac{1}{\cos^2 45^\circ}$

$= \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}$

$= \frac{1}{\frac{2}{4}} = \frac{1}{\frac{1}{2}} = 2$

Linear Appr. near $x = a$

$f(x) \approx f(a) + f'(a)(x - a)$

$\tan x \approx 1 + 2\left(x - \frac{\pi}{4}\right)$

$\tan 46^\circ \approx 1 + 2(46^\circ - 45^\circ)$

$= 1 + 2 \cdot 1^\circ$

$= 1 + 2 \cdot \frac{\pi}{180} = 1 + \frac{\pi}{90}$

$= \frac{90 + \pi}{90}$

$\approx \boxed{1.035}$

Mar 19-9:36 AM

Estimate $\frac{1}{4.002}$ using linear approximation.

$\frac{1}{4.002} \approx \frac{1}{4} \approx .25$

a

$f(x) = \frac{1}{x}$

$a = 4$

$f(a) = \frac{1}{4} = .25$

$f'(x) = \frac{-1}{x^2}$

$f'(a) = \frac{-1}{4^2} = \frac{-1}{16}$

Use your calc to find $\frac{1}{4.002}$

$= .249875\,0625\ldots$

$f(x) \approx f(a) + f'(a)(x - a)$

$\frac{1}{x} \approx \frac{1}{4} + \frac{-1}{16}(x - 4)$

at 4.002

$\frac{1}{4.002} \approx \frac{1}{4} - \frac{1}{16}(4.002 - 4)$

$= \frac{1}{4} - \frac{1}{16}(.002)$

$= \frac{1}{4} - \frac{1}{\cancel{16}\,8} \cdot \frac{\cancel{2}}{1000}$

$= \frac{1}{4} - \frac{1}{8000}$

$= \frac{1 \cdot 2000}{4 \cdot 2000} - \frac{1}{8000}$

$= \frac{2000}{8000} - \frac{1}{8000}$

$= \frac{1999}{8000} \approx .249875$

Mar 19-9:46 AM