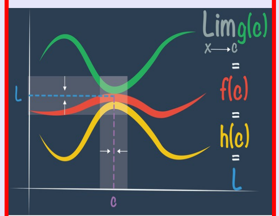
Calculus I

Lecture 23

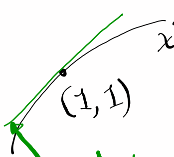


Feb 19-8:47 AM

Class QZ 13

Given $\boxed{x^2 + xy + y^2 = 3}$

Find $\left.\frac{dy}{dx}\right|_{(1,1)}$



$x^2+xy+y^2=3$

$1^2+1\cdot1+1^2=3$

$3=3$

$(1,1)$

$m = \left.\frac{dy}{dx}\right|_{(1,1)} = -1$

$y-1 = -1(x-1)$

$y = -x+1+1$

$\boxed{y=-x+2}$

$\frac{d}{dx}[x^2+xy+y^2] = \frac{d}{dx}[3]$

$\frac{d}{dx}[x^2] + \frac{d}{dx}[xy] + \frac{d}{dx}[y^2] = 0$

$2x + 1\cdot y + x\cdot\frac{dy}{dx} + 2y\cdot\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$

$(x+2y)\boxed{\frac{dy}{dx}} = -2x-y$

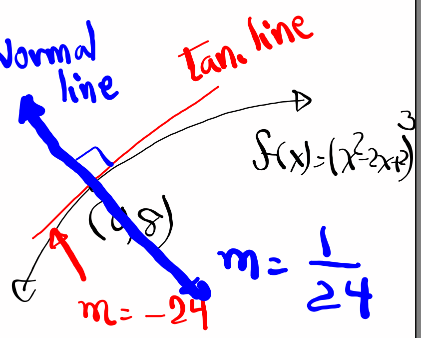
$\boxed{\frac{dy}{dx} = \frac{-2x-y}{x+2y}}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2\cdot1-1}{1+2\cdot1} = \frac{-3}{3}$

$= \boxed{-1}$ ✓

Mar 14-9:45 AM

Given $f(x) = (x^2 - 2x + 2)^3$



$f(0) = (0^2 - 2(0) + 2)^3 = 2^3 = \boxed{8}$

$f'(x) = 3(x^2 - 2x + 2)^2 \cdot (2x - 2)$

$f'(0) = 3(0^2 - 2(0) + 2)^2 \cdot (2 \cdot 0 - 2) = 3 \cdot 2^2 \cdot (-2) = 3 \cdot 4 \cdot -2 = \boxed{-24}$

Now eqn of tan. line $y - 8 = -24(x - 0)$

and " " Normal line $y - 8 = \frac{1}{24}(x - 0)$

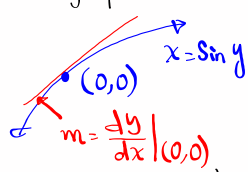
Mar 18-4:02 PM

Given $x = \sin y$

1) verify that $(0,0)$ is on the graph of $x = \sin y$.

$0 = \sin 0$

$0 = 0 \checkmark$



2) find slope of the tan. line to the graph of $x = \sin y$ at $(0,0)$.

$\frac{d}{dx}[x] = \frac{d}{dx}[\sin y]$

$1 = \cos y \cdot \frac{dy}{dx} \to \frac{dy}{dx} = \frac{1}{\cos y}$

$m = \frac{dy}{dx}\Big|_{(0,0)} = \frac{1}{\cos 0} = \frac{1}{1} = \boxed{1}$

what about eqn of tan. line at $(0,0)$?

$y - 0 = 1(x - 0)$

$\boxed{y = x}$

Mar 18-4:07 PM

$$x^3 + y^2 = 5$$

Find $\frac{dy}{dx}$

$$\frac{d}{dx}[x^3] + \frac{d}{dx}[y^2] = \frac{d}{dx}[5]$$

$$3x^2 + 2y \cdot \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2}{2y}}$$

Mar 18-4:13 PM

Given $\tan\left(\frac{x}{y}\right) = x^2 + y^2$, Find $\frac{dy}{dx}$

$$\frac{d}{dx}\left[\tan \frac{x}{y}\right] = \frac{d}{dx}[x^2] + \frac{d}{dx}[y^2]$$

$$\sec^2 \frac{x}{y} \cdot \frac{1 \cdot y - x \cdot \frac{dy}{dx}}{y^2} = 2x + 2y \cdot \frac{dy}{dx}$$

Multiply by y^2

$$\sec^2 \frac{x}{y} \cdot \left(y - x \cdot \frac{dy}{dx}\right) = 2xy^2 + 2y^3 \frac{dy}{dx}$$

$$y \sec^2 \frac{x}{y} \left(- x \sec^2 \frac{x}{y} \cdot \frac{dy}{dx}\right) = 2xy^2 + 2y^3 \frac{dy}{dx}$$

$$y \sec^2 \frac{x}{y} - 2xy^2 = x \sec^2 \frac{x}{y} \cdot \frac{dy}{dx} + 2y^3 \cdot \frac{dy}{dx}$$

$$y \sec^2 \frac{x}{y} - 2xy^2 = \left(x \sec^2 \frac{x}{y} + 2y^3\right) \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{y \sec^2 \frac{x}{y} - 2xy^2}{x \sec^2 \frac{x}{y} + 2y^3}}$$

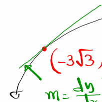
Mar 18-4:15 PM

Given $\sqrt[3]{x^2} + \sqrt[3]{y^2} = 4$ ✓

1) verify $(-3\sqrt{3}, 1)$ is on the graph of the given equation.

$$\sqrt[3]{(-3\sqrt{3})^2} + \sqrt[3]{1^2} = \sqrt[3]{9 \cdot 3} + \sqrt[3]{1}$$
$$= \sqrt[3]{27} + \sqrt[3]{1} = 3 + 1 = 4 \checkmark$$

2) find eqn. of tan line at the given Point to the graph of the given eqn.



$\sqrt[3]{x^2} + \sqrt[3]{y^2} = 4$

$(-3\sqrt{3}, 1)$

$m = \left.\frac{dy}{dx}\right|_{(-3\sqrt{3}, 1)}$

$$\sqrt[3]{x^2} + \sqrt[3]{y^2} = 4$$
$$x^{2/3} + y^{2/3} = 4$$
$$\frac{2}{3} x^{\frac{2}{3}-1} + \frac{2}{3} y^{\frac{2}{3}-1} \cdot \frac{dy}{dx} = 0$$
$$\frac{2}{3} x^{-\frac{1}{3}} + \frac{2}{3} y^{-\frac{1}{3}} \frac{dy}{dx} = 0$$

Multiply by $\frac{3}{2}$

$$\cancel{\tfrac{3}{2}} \cdot \cancel{\tfrac{2}{3}} x^{-1/3} + \cancel{\tfrac{3}{2}} \cdot \cancel{\tfrac{2}{3}} \cdot y^{-1/3} \cdot \frac{dy}{dx} = 0$$
$$x^{-1/3} + y^{-1/3} \frac{dy}{dx} = 0$$

$$y^{-1/3} \frac{dy}{dx} = -x^{-1/3}$$
$$\frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}}$$
$$= -\frac{y^{1/3}}{x^{1/3}}$$
$$\frac{dy}{dx} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

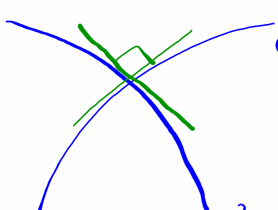
$$m = -\frac{\sqrt[3]{1}}{\sqrt[3]{-3\sqrt{3}}} = \frac{1}{\sqrt[3]{3\sqrt{3}}} = \frac{1}{\sqrt[3]{3\sqrt{3}}}$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{1}{\sqrt[3]{3\sqrt{3}}}(x - -3\sqrt{3})$$
$$y - 1 = \frac{1}{\sqrt[3]{3\sqrt{3}}} x + \frac{3\sqrt{3}}{\sqrt[3]{3\sqrt{3}}} \Rightarrow \boxed{y = \frac{1}{\sqrt[3]{3\sqrt{3}}} x + \frac{3\sqrt{3}}{\sqrt[3]{3\sqrt{3}}} + 1}$$

Mar 18-4:23 PM

Show $y = cx^2$ and $x^2 + 2y^2 = K$ are orthogonal curves.

their tan. lines at intersections are Perpendicular to each other.



$y = cx^2 \rightarrow y' = 2cx$

$x^2 + 2y^2 = K \rightarrow 2x + 4y \cdot y' = 0$

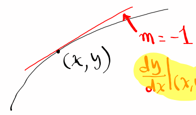
$$y' = \frac{-2x}{4y} = \frac{-x}{2y}$$

we need to product of derivatives is -1

$$\cancel{2}cx \cdot \frac{-x}{\cancel{2}y} = \frac{-cx^2}{y} = \frac{-y}{y} = \boxed{-1}$$

Mar 18-4:38 PM

Find all points on the curve $x^2y^2 + xy = 2$ where slope of tangent line is -1.



$\left.\frac{dy}{dx}\right|_{(x,y)} = -1$

$x^2y^2 + xy = 2$

$2x \cdot y^2 + x^2 \cdot 2y \cdot \frac{dy}{dx} + 1 \cdot y + x \cdot \frac{dy}{dx} = 0$

$2xy^2 + 2x^2y(-1) + y + x(-1) = 0$

$2xy^2 - 2x^2y + y - x = 0$

$2xy(y-x) + 1(y-x) = 0$

$(y-x)(2xy+1) = 0$

$y - x = 0$ $\qquad$ $2xy + 1 = 0$

$y = x$ $\qquad$ $y = \frac{-1}{2x}$

$x^2y^2 + xy = 2$

$x^2x^2 + xx = 2$

$x^4 + x^2 - 2 = 0$

$(x^2+2)(x^2-1) = 0$

$x^2 + 2 \neq 0$ $\qquad$ $x^2 - 1 = 0$

$x = 1,\ x = -1$

$x^2y^2 + xy = 2$

$x^2 \cdot \left(\frac{-1}{2x}\right)^2 + x \cdot \frac{-1}{2x} = 2$

$\frac{x^2}{4x^2} - \frac{x}{2x} = 2$

$\frac{1}{4} - \frac{1}{2} \neq 2$

Points are

$(1, 1), (-1, -1)$

Mar 18-4:44 PM