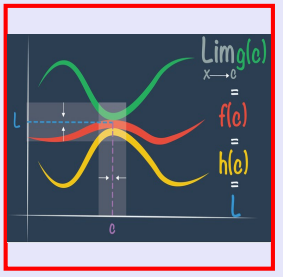
Calculus I

Lecture 22



Feb 19-8:47 AM

Class Qz 12

Find $\frac{dy}{dx}$, Do not simplify

1) $y = \frac{1}{2}\tan(x^2)$

$\frac{dy}{dx} = \frac{1}{2} \cdot \sec^2(x^2) \cdot 2x$

2) $y = (x^3 - x)^5$

$\frac{dy}{dx} = 5(x^3 - x)^4 \cdot (3x^2 - 1)$

Mar 14-7:55 AM

Given $y = \sqrt{x}$

$y = x^{1/2}$ → $y' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{1/2}}$

$y' = \frac{1}{2\sqrt{x}}$

If we square both sides

$y^2 = x$

Let's take the derivative of both sides with respect to x.

$y = f(x)$

Implicit Differentiation

$\frac{d}{dx}[y^2] = \frac{d}{dx}[x]$

$2y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$

$\boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}}}$

Mar 14-9:04 AM

Given $y = \frac{1}{x}$

$y = x^{-1}$

$y' = -1 \cdot x^{-2}$

$\boxed{y' = \frac{-1}{x^2}}$

multiply by x

$xy = 1$

Differentiate both sides with respect to x.

Implicit Diff.

$\frac{d}{dx}[xy] = \frac{d}{dx}[1]$

Product Rule

$\frac{d}{dx}[x] \cdot y + x \cdot \frac{d}{dx}[y] = 0$

$1 \cdot y + x \cdot \frac{dy}{dx} = 0$

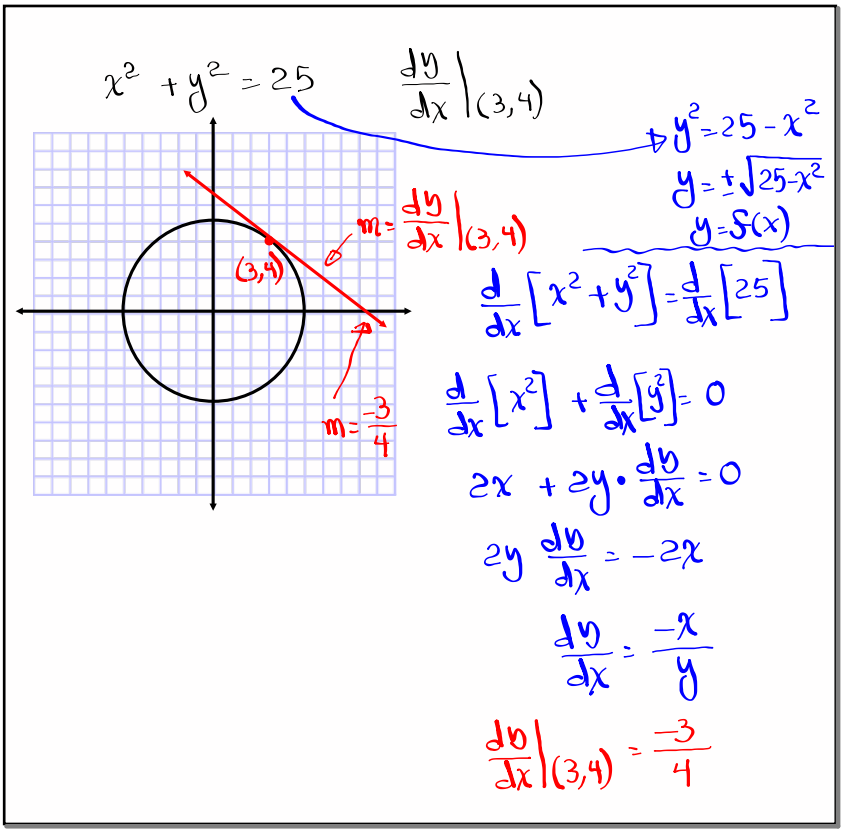
$x\frac{dy}{dx} = -y$

$\frac{dy}{dx} = \frac{-y}{x}$

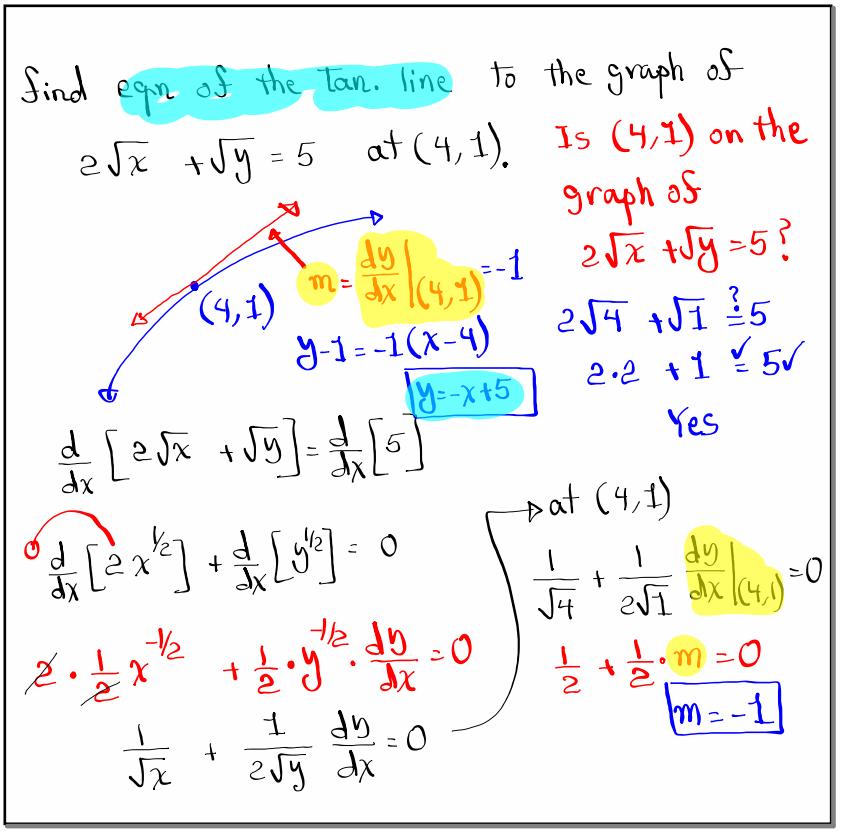
$\frac{dy}{dx} = \frac{-\frac{1}{x}}{x}$

$\boxed{\frac{dy}{dx} = \frac{-1}{x^2}}$

Mar 14-9:09 AM



Mar 14-9:15 AM



Mar 14-9:21 AM

Given $x^2y^3 = x + 3$

Find $\frac{dy}{dx}$

$$\frac{d}{dx}\left[x^2y^3\right] = \frac{d}{dx}\left[x+3\right]$$

Product

$$\frac{d}{dx}[x^2]\cdot y^3 + x^2\cdot\frac{d}{dx}[y^3] = \frac{d}{dx}[x] + \frac{d}{dx}[3]$$

$$2x\cdot y^3 + x^2\cdot 3y^2\cdot\frac{dy}{dx} = 1 + 0$$

$$3x^2y^2\frac{dy}{dx} = 1 - 2xy^3$$

$$\boxed{\frac{dy}{dx} = \frac{1-2xy^3}{3x^2y^2}}$$

Mar 14-9:30 AM

Find y'' for $x^3 + y^3 = 1$.

$$\frac{d}{dx}[x^3] + \frac{d}{dx}[y^3] = \frac{d}{dx}[1]$$

$$3x^2 + 3y^2\cdot y' = 0$$

$$\boxed{y' = \frac{-x^2}{y^2}}$$

$\frac{d}{dx}[-x^2]$

$$y'' = \frac{-2x\cdot y^2 - (-x^2)\cdot\frac{d}{dx}[y^2]}{(y^2)^2} = \frac{-2xy^2 + x^2\cdot 2y\cdot\frac{dy}{dx}}{y^4}$$

$$= \frac{-2xy^2 + 2x^2y\cdot\frac{-x^2}{y^2}}{y^4}$$

LCD y

$$= \frac{-2xy^3 - 2x^4}{y^5}$$

$$= \frac{-2x(y^3 + x^3)}{y^5}$$

$$= \frac{-2x\cdot 1}{y^5} = \boxed{\frac{-2x}{y^5}}$$

$$y'' = \frac{-2x}{y^5}$$

Mar 14-9:36 AM

Class QZ 13

Given $x^2 + xy + y^2 = 3$

Find $\left.\frac{dy}{dx}\right|_{(1,1)}$

$$\frac{d}{dx}\left[x^2 + xy + y^2\right] = \frac{d}{dx}[3]$$

$$\frac{d}{dx}[x^2] + \frac{d}{dx}[xy] + \frac{d}{dx}[y^2] = 0$$

$$2x + 1 \cdot y + x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$(x + 2y)\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2\cdot 1 - 1}{1 + 2\cdot 1}$$

$$= \boxed{-1}$$

Mar 14-9:45 AM