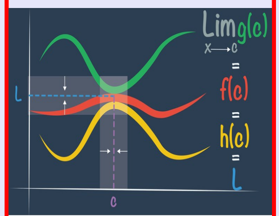
Calculus I

Lecture 21



Feb 19-8:47 AM

Class QZ 11

Given $y = \sin u$, $u = \frac{1}{x^2}$ Find $\left.\frac{dy}{dx}\right|_{x=1}$

$\boxed{u = x^{-2}}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \cos \boxed{u} \cdot \boxed{-2x^{-3}} \rightarrow \frac{dy}{dx} = \boxed{\cos \boxed{\frac{1}{x^2}} \cdot \boxed{\frac{-2}{x^3}}}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \boxed{-2\cos 1} \approx \boxed{-1.081}$$

Mar 12-9:48 AM

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If I let $a + h = x$

$h = x - a$

as $h \to 0$

$x \to a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.

$$f(x) - f(a) = \frac{f(x) - f(a)}{x - a} \cdot (x - a)$$

$$\lim_{x \to a} \left[f(x) - f(a)\right] = \lim_{x \to a} \left[\frac{f(x) - f(a)}{x - a} \cdot (x - a)\right]$$

$$= \boxed{\lim_{x \to a} \frac{f(x) - f(a)}{x - a}} \cdot \boxed{\lim_{x \to a} (x - a)}$$

$$= f'(a) \cdot (\underbrace{a - a}_{\to 0})$$

$$= f'(a) \cdot 0 = 0$$

So $\lim_{x \to a} \left[f(x) - f(a)\right] = 0$

$\lim_{x \to a} f(x) - \lim_{x \to a} f(a) = 0$

$\lim_{x \to a} f(x) - f(a) = 0 \to \lim_{x \to a} f(x) = f(a)$

$\therefore f(x)$ is cont. at $x = a$.

Mar 13-8:50 AM

More on chain rule

Recall from Pre Calc $\to (f \circ g)(x) = f(g(x))$

$f(x) = \sin x \quad , \quad g(x) = x^3$

$$(f \circ g)(x) = f(g(x)) = \sin g(x) = \sin(x^3)$$

$f(x) = x^4 \quad , \quad g(x) = \cos(x-1)$

$$(f \circ g)(x) = f(g(x)) = (g(x))^4 = [\cos(x-1)]^4$$

$$= \cos^4(x-1)$$

$$\frac{d}{dx}\left[(f \circ g)(x)\right] = \frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}\left[\sin(x^2 - 4x)\right] = \boxed{\cos(x^2 - 4x) \cdot (2x - 4)}$$

Mar 13-8:59 AM

$f(x) = (x^3 + 5x^2)^{10}$ $\quad u^{10}$

$$\boxed{f'(x) = 10(x^3+5x^2)^9 \cdot (3x^2 + 10x)}$$

$f(x) = \sin^4(\sqrt{x})$ $\quad f(x) = [\sin\sqrt{x}]^4$ $\quad u^4$

$f'(x) = \cancel{4}^{\,2}[\sin\sqrt{x}]^3 \cdot \cos\sqrt{x} \cdot \frac{1}{\cancel{2}} x^{-1/2}$

$\frac{d}{dx}[\sqrt{x}] =$

$\frac{d}{dx}[x^{1/2}]$

$= \dfrac{2\sin^3\sqrt{x} \cdot \cos\sqrt{x}}{x^{1/2}}$

$$\boxed{f'(x) = \frac{2\sin^3\sqrt{x} \cdot \cos\sqrt{x}}{\sqrt{x}}}$$

Mar 13-9:06 AM

$f(x) = \dfrac{1}{\sqrt[4]{x^2+2x+2}}$

$f(-1) = \dfrac{1}{\sqrt[4]{(-1)^2+2(-1)+2}}$

$f(x) = (x^2+2x+2)^{\boxed{-\frac{1}{4}}}$

$= \dfrac{1}{\sqrt[4]{1-2+2}}$

$f'(x) = \frac{-1}{4}(x^2+2x+2)^{-\frac{1}{4}-1} \cdot (2x+2)$

$= \dfrac{1}{\sqrt[4]{1}} = \boxed{1}$

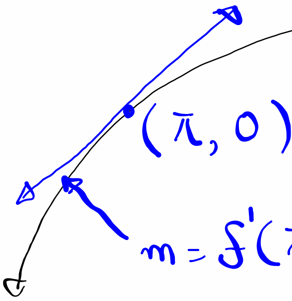
$= \dfrac{-2(x+1)}{4(x^2+2x+2)^{5/4}} = \dfrac{-(x+1)}{2(x^2+2x+2)^{4/4}\cdot(x^2+2x+2)^{1/4}}$

$= \dfrac{-(x+1)}{2(x^2+2x+2)\cdot\sqrt[4]{x^2+2x+2}}$

Mar 13-9:13 AM

Find eqn of tan. line to the graph of

$f(x) = \sin(\sin x)$ at $x = \pi$.



$\to f(x) = \sin(\sin x)$

$f(\pi) = \sin(\cancel{\sin \pi}^{\,0})$

$= \sin 0$

$= 0$

$m = f'(\pi) = -1$

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x - \pi)$

$\boxed{y = -x + \pi}$

$f(x) = \sin(\sin x)$

$f'(x) = \cos(\sin x) \cdot \cos x$

$f'(\pi) = \cos(\cancel{\sin \pi}^{\,0}) \cdot \cos \pi = \cancel{\cos 0}^{\,1} \cdot \cancel{\cos \pi}^{\,-1} = 1 \cdot (-1) = \boxed{-1}$

Mar 13-9:20 AM

Given $f(x) = (2x-3)^4 \cdot (4x^2 + 6x + 9)^4$

Find $f'(2)$

$f(x) = \left[(2x-3)(4x^2+6x+9)\right]^4$

$f(x) = \left[8x^3 - 27\right]^{4}$

$f'(x) = 4\left[8x^3 - 27\right]^3 \cdot 24x^2$

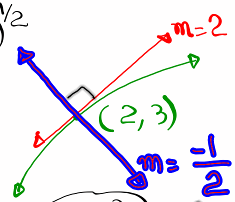
$f'(x) = 96x^2(8x^3 - 27)^3$

$f'(2) = 96(2)^2\left[8 \cdot 2^3 - 27\right]^3$

$\boxed{f'(2) = 19450752}$

Mar 13-9:26 AM

$f(x)=\sqrt{1+x^3}$ $\qquad f(x)=(1+x^3)^{1/2}$



1) Find $f(2)=\sqrt{1+2^3}=\sqrt{9}=\boxed{3}$

2) Find $f'(x)=\frac{1}{2}(1+x^3)^{\frac{1}{2}-1}\cdot 3x^2=\frac{3x^2}{2\sqrt{1+x^3}}$

3) Find $f'(2)=\frac{3(2)^2}{2\sqrt{1+2^3}}=\frac{3\cdot 4}{2\cdot 3}=\boxed{2}$

4) Find eqn of the normal line to the graph of $f(x)$ at $x=2$.

$y-y_1=m(x-x_1)$

$y-3=\frac{-1}{2}(x-2)$ $\qquad y-3=\frac{-1}{2}x+1$

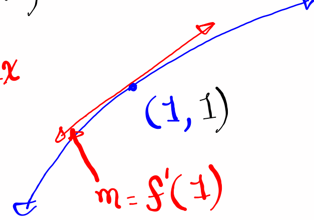
$\boxed{y=\frac{-1}{2}x+4}$

Mar 13-9:32 AM

find eqn of the tan. line to the graph of

$f(x)=\tan\left(\frac{\pi}{4}x^2\right)$ at $x=1$.

$f(x)=\tan\left(\frac{\pi}{4}x^2\right)$



$f(1)=\tan\left(\frac{\pi}{4}\cdot 1^2\right)=\tan\frac{\pi}{4}=1$

$f'(x)=\sec^2\left(\frac{\pi}{4}x^2\right)\cdot\frac{\pi}{4}\cdot 2x$

$f'(x)=\frac{\pi}{2}\cdot x\cdot\sec^2\left(\frac{\pi}{4}x^2\right)$

$m=f'(1)=\frac{\pi}{2}\cdot 1\cdot\sec^2\left(\frac{\pi}{4}\cdot 1^2\right)=\frac{\pi}{2}\cdot\left[\sec\frac{\pi}{4}\right]^2=\frac{\pi}{2}\cdot(\sqrt{2})^2=\pi$

$y-y_1=m(x-x_1)$

$y-1=\pi(x-1)$

$\boxed{y=\pi x-\pi+1}$

$m=\pi$

Y-Int $(0,-\pi+1)$

Mar 13-9:45 AM