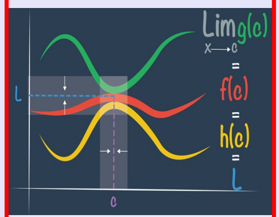
Calculus I

Lecture 19



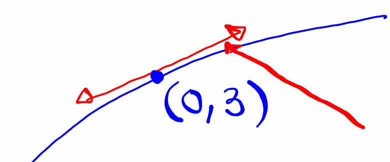
Feb 19-8:47 AM

Class QZ 9

Find $\boxed{\text{eqn of the tan. line}}$ to the graph of

$\boxed{f(x) = x^3 - 2x + 3}$ $\boxed{\text{at } x = 0}$

$f(0) = 0^3 - 2(0) + 3$
$= 3$



$\boxed{f'(x) = 3x^2 - 2}$

$m = f'(0) = -2$ ✓

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - 0) \rightarrow \boxed{y = -2x + 3}$ ✓

Mar 7-9:40 AM

What we know so far

1) $\frac{d}{dx}[c]=0$

2) $\frac{d}{dx}[cf(x)]=cf'(x)$

3) $\frac{d}{dx}[f(x)\pm g(x)]=f'(x)\pm g'(x)$

4) $\frac{d}{dx}[f(x)\cdot g(x)]=f'(x)\cdot g(x)+f(x)\cdot g'(x)$

5) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{[g(x)]^2}$

6) $\frac{d}{dx}[x^n]=nx^{n-1}$

7) $\frac{d}{dx}[\sin x]=\cos x$ $\quad$ $\frac{d}{dx}[\csc x]=-\csc x\cot x$

$\frac{d}{dx}[\cos x]=-\sin x$ $\quad$ $\frac{d}{dx}[\sec x]=\sec x\tan x$

$\frac{d}{dx}[\tan x]=\sec^2 x$ $\quad$ $\frac{d}{dx}[\cot x]=-\csc^2 x$

Mar 11-8:47 AM

Show $\frac{d}{dx}[\csc x]=-\csc x\cot x$ ✓

$$\frac{d}{dx}[\csc x]=\frac{d}{dx}\left[\frac{1}{\sin x}\right]=\frac{\cancel{\frac{d}{dx}[1]}^{\,0}\cdot\sin x-1\cdot\frac{d}{dx}[\sin x]}{(\sin x)^2}$$

$$=\frac{\cancel{0\cdot\sin x}^{\,0}-1\cdot\cos x}{\sin^2 x}$$

$$=\frac{-\cos x}{\sin^2 x}=-\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}$$

$$=\boxed{-\csc x\cot x}$$

Mar 11-8:54 AM

Given $f(x) = \frac{x^2}{x-1}$

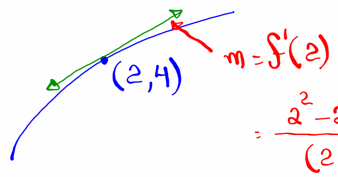
Domain $\to x \neq 1$ Y-Int $(0,0)$

V.A. $\to x = 1$ x-Int $(0,0)$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2}{x-1} = \infty$, $\lim_{x\to-\infty} f(x) = -\infty$

Find eqn of tan. line to the graph of $f(x) = \frac{x^2}{x-1}$ at $x = 2$.

$f(2) = \frac{2^2}{2-1} = 4$



$f(x) = \frac{x^2}{x-1}$

$f'(x) = \frac{2x(x-1) - x^2 \cdot 1}{(x-1)^2}$

$= \frac{2x^2 - 2x - x^2}{(x-1)^2}$

$= \frac{x^2 - 2x}{(x-1)^2}$

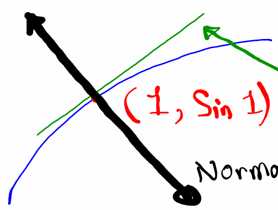
$m = f'(2) = \frac{2^2 - 2(2)}{(2-1)^2} = \frac{4-4}{1^2} = \boxed{0}$

$y - 4 = 0(x-2)$

$y - 4 = 0$ $\boxed{y = 4}$

Mar 11-8:58 AM

Find the eqn of the normal line to the graph of $f(x) = x^2 \sin x$ at $x = 1$.



$f(x) = x^2 \sin x$ $f(1) = 1^2 \cdot \sin 1 = \sin 1$

tan. line $\to m = f'(1)$

Normal line $\to m = \frac{-1}{f'(1)} = \frac{-1}{2\sin 1 + \cos 1} \approx \boxed{-.45}$

we need $f'(x)$

$f(x) = x^2 \sin x$

$f'(x) = 2x \cdot \sin x + x^2 \cdot \cos x$

$f'(1) = 2\sin 1 + \cos 1$

$y - y_1 = m(x - x_1)$

$y - \sin 1 \approx -.45(x-1)$

$y \approx -.45x + .45 + \sin 1$

$\boxed{y \approx -.45x + 1.29}$

Mar 11-9:05 AM

Given $y = 5x^2 - 4x + 7$ Find

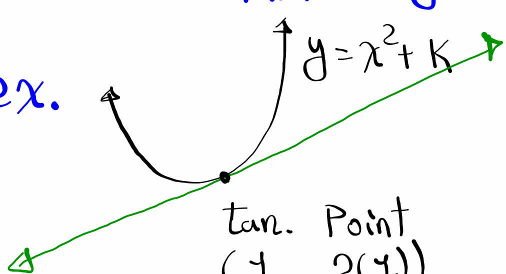
$$\frac{dy}{dx} = y' = 10x - 4$$

$$\frac{d^2y}{dx^2} = y'' = \frac{d}{dx}\left[y'\right] = \frac{d}{dx}\left[10x - 4\right] = 10$$

$$\frac{d^3y}{dx^3} = y''' = \frac{d}{dx}\left[y''\right] = \frac{d}{dx}\left[10\right] = 0$$

Mar 11-9:18 AM

Find K Such that $y = x^2 + K$ is tan. to $y = 2x$.



$$\frac{d}{dx}\left[x^2 + K\right] = \frac{d}{dx}\left[2x\right]$$

$$2x = 2$$

$$x = 1$$

tan. Point
$(1, 2(1))$

$(1, 1^2 + K)$

$1^2 + K = 2(1)$

$1 + K = 2$

$\boxed{K = 1}$

Mar 11-9:22 AM

find points, if there is any, on the graph of

$f(x) = \dfrac{x}{x^2+9}$ where tan. line is horizontal.

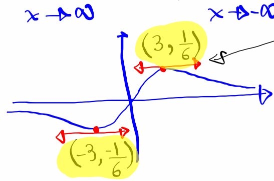
$f(-x) = \dfrac{-x}{(-x)^2+9} = \dfrac{-x}{x^2+9} = -\dfrac{x}{x^2+9} = -f(x)$

$f(x)$ is an odd function $\to$ Symmetric $\to$ origin

$x^2+9 \neq 0 \to$ Domain is $(-\infty, \infty)$ No V.A.

x-Int, y-Int $(0,0)$

$\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = 0 \to$ H.A. $\to y=0$



$f'(x) = 0$

$f(x) = \dfrac{x}{x^2+9}$

$f'(x) = \dfrac{1\cdot(x^2+9) - x\cdot 2x}{(x^2+9)^2}$

$= \dfrac{9-x^2}{(x^2+9)^2}$

Solve $f'(x) = 0$

$\dfrac{9-x^2}{(x^2+9)^2} = 0 \to 9 - x^2 = 0$

$x = \pm 3$

$f(3) = \dfrac{3}{3^2+9} = \dfrac{3}{18} = \dfrac{1}{6}$

Mar 11-9:27 AM

If $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.

Look it up.

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find $\left.\dfrac{d^2y}{dx^2}\right|_{x=\frac{\pi}{4}}$ if $y = \sec x$ $\to y''\left(\frac{\pi}{4}\right)$

$y = \sec x$ $\quad y' = \boxed{\sec x}\cdot\boxed{\tan x}$

$y'' = \boxed{\sec x \tan x}\cdot \tan x + \sec x \cdot \boxed{\sec^2 x}$

$y''\left(\frac{\pi}{4}\right) = \sec\frac{\pi}{4}\cdot\tan\frac{\pi}{4}\cdot\tan\frac{\pi}{4} + \left(\sec\frac{\pi}{4}\right)^3$

$= \sqrt{2}\cdot 1 \cdot 1 + (\sqrt{2})^3$

$= \sqrt{2} + 2\sqrt{2} = \boxed{3\sqrt{2}}$

Mar 11-9:39 AM

Class QZ 10

$$y = ax^3 + bx^2 + cx + d$$

Find y', y'', and y'''. Make sure to box your answers.

$$\boxed{y' = 3ax^2 + 2bx + c}$$

$$\boxed{y'' = 6ax + 2b}$$

$$\boxed{y''' = 6a}$$

Mar 11-9:47 AM