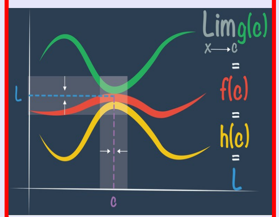
Calculus I

Lecture 18



Feb 19-8:47 AM

Find the equation of the tan. line to the graph of

$f(x)=\frac{-1}{x^2}$ at $x=1$. $\quad f(1)=\frac{-1}{(1)^2}=-1$

Domain: $x\neq 0$

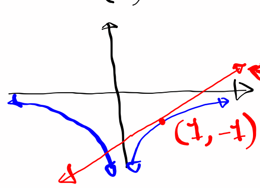
V.A. $x=0$

H.A. $y=0$ → $\lim_{x\to\infty} f(x)=0$, $\lim_{x\to-\infty} f(x)=0$

$\frac{-1<0}{x^2>0}$ → $f(x)<0$ → $y<0$ → Graph is below x-axis

$f(-x)=\frac{-1}{(-x)^2}=\frac{-1}{x^2}=f(x)$ → $f(-x)=f(x)$ → even function

→ symmetric with respect to Y-axis



$m=f'(1)$

$=\frac{2}{1^3}=\boxed{2}$

$f(x)=\frac{-1}{x^2}=-1x^{-2}=-x^{-2}$ → $f'(x)=-\cdot(-2)x^{-2-1}$

Power rule

$f'(x)=2x^{-3}=\frac{2}{x^3}$

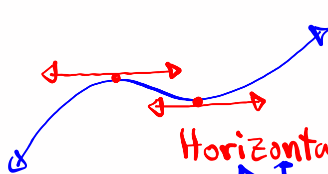
$y-(-1)=2(x-1)$

$\boxed{y=2x-3}$

Mar 7-7:55 AM

$(\pi, f(\pi)) = (\pi, \pi)$ $f(\pi) = \cancel{\sin\pi}^{\,0} + \pi$

Find all points on the graph of $f(x) = \sin x + x$ where we have horizontal tan. line on $[0, 2\pi]$.



Suppose $f(x) = \sin x + x$

Horizontal tan. line

$m = 0$ $f'(x) = 0$

$f(x) = \sin x + x$

$f'(x) = \cos x + 1$

Solve

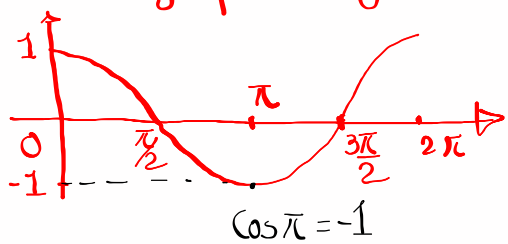
$\cos x + 1 = 0$

$\cos x = -1$

$\boxed{x = \pi}$

$f'(\pi) = 0$

Recall the graph of $y = \cos x$



$\cos\pi = -1$

Mar 7-8:56 AM

Find all points on the graph of any Quadratic Function where we have horizontal tan. line.

$f(x) = ax^2 + bx + c$, $a \neq 0$

$f'(x) = 0$ $f'(x) = 2ax + b$

$2ax + b = 0$

$2ax = -b$

$x = \frac{-b}{2a}$

For points, we need $f\left(\frac{-b}{2a}\right) = a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c$

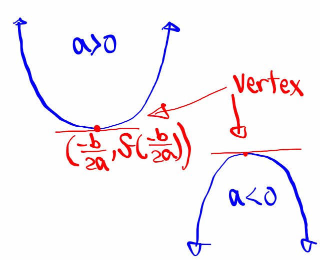
$= \cancel{a} \cdot \frac{b^2}{4a^{\cancel{2}}} - \frac{b^2}{2a} + c$

$= \frac{b^2}{4a} - \frac{b^2}{2a} + c$

$= \frac{b^2 - 2b^2 + 4ac}{4a}$

$= \frac{4ac - b^2}{4a}$

$f(x) = ax^2 + bx + c$, $a \neq 0$



Horizontal tan. line happens at $\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$

Mar 7-9:04 AM

$f(x) = \dfrac{x}{x^2-1}$

$f(-x) = \dfrac{-x}{(-x)^2-1} = \dfrac{-x}{x^2-1} = -\dfrac{x}{x^2-1} = -f(x)$

$f(0) = \dfrac{0}{0^2-1} = 0$

$f(-x) = -f(x)$

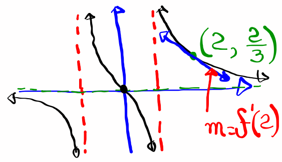
$f(x)$ is odd, Sym. w/t origin

Domain $\rightarrow x \neq \pm 1$

V.A. at $x = \pm 1$

H.A. at $y = 0$

$\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to -\infty} f(x) = 0$



$f(2) = \dfrac{2}{2^2-1} = \dfrac{2}{3}$

Find eqn of tan. line to the graph of $f(x)$ at $x = 2$

$f(x) = \dfrac{x}{x^2-1}$ $\quad f'(x) = \dfrac{1 \cdot (x^2-1) - x \cdot 2x}{(x^2-1)^2} = \dfrac{x^2-1-2x^2}{(x^2-1)^2}$

Quotient Rule

$= \dfrac{-x^2-1}{(x^2-1)^2}$

$f'(2) = \dfrac{-(2)^2-1}{(2^2-1)^2} = \dfrac{-4-1}{3^2} = \boxed{\dfrac{-5}{9}}$

tan. Point $(2, \frac{2}{3})$

$m_{\text{tan. line}} = f'(2) = \frac{-5}{9}$

$y - \frac{2}{3} = \frac{-5}{9}(x-2)$

$\vdots$

$\boxed{y = }$

Finish it on your own time

Mar 7-9:14 AM

Find $\dfrac{d}{dx}[\tan x]$

$\dfrac{d}{dx}[\tan x] = \dfrac{d}{dx}\left[\dfrac{\sin x}{\cos x}\right] = \dfrac{\frac{d}{dx}[\sin x] \cdot \cos x - \sin x \cdot \frac{d}{dx}[\cos x]}{(\cos x)^2}$

Quotient Rule

$\dfrac{d}{dx}[\sin x] = \cos x$

$\dfrac{d}{dx}[\cos x] = -\sin x$

$\dfrac{d}{dx}[\tan x] = \sec^2 x$

$= \dfrac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x} = \boxed{\sec^2 x}$

$\dfrac{d}{dx}[\sec x]$

$\dfrac{d}{dx}[\csc x]$

$\dfrac{d}{dx}[\cot x]$

Look these up

Mar 7-9:26 AM

Find eqn of the normal line to the graph of

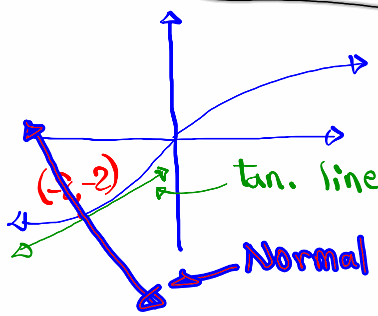
$f(x) = \sqrt[3]{x}$ at $x = -8$. $f(-8) = \sqrt[3]{-8} = -2$

$\to f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1}$

$= \frac{1}{3}x^{-2/3}$

$= \frac{1}{3\sqrt[3]{x^2}}$



tan. line $m = f'(-8)$

Normal line $m = \dfrac{-1}{f'(-8)}$

$f'(-8) = \dfrac{1}{3\sqrt[3]{(-8)^2}} = \dfrac{1}{3\sqrt[3]{64}} = \dfrac{1}{3 \cdot 4} = \dfrac{1}{12}$

$m_{\text{Normal Line}} = \dfrac{-1}{\frac{1}{12}} = \boxed{-12}$

Normal line

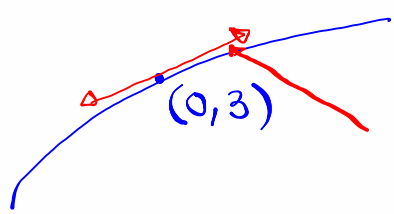
$y-(-2) = -12(x-(-8)) \to \boxed{y = \quad}$ finish it off

Mar 7-9:32 AM

Class QZ 9

Find eqn of the tan. line to the graph of

$f(x) = x^3 - 2x + 3$ at $x = 0$. $f(0) = 0^3 - 2(0) + 3$
$= 3$



$f'(x) = 3x^2 - 2$

$m = f'(0) = -2$

$y - 3 = -2(x-0) \to \boxed{y = -2x + 3}$

Mar 7-9:40 AM