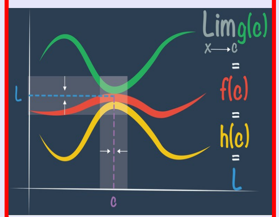
# Calculus I

# Lecture 17




Feb 19-8:47 AM

Given $f(x) = x^3 - x$

1) Y-Int. $\Rightarrow x = 0 \Rightarrow f(0) = 0^3 - 0 = 0 \Rightarrow (0, 0)$

2) x-Int. $\Rightarrow y = 0 \Rightarrow f(x) = 0 \Rightarrow x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$0 \quad -1 \quad 1$

$\Rightarrow (0,0), (-1,0), (1,0)$

3) Is $f(x)$ even, odd, or neither?

$f(-x) = (-x)^3 - (-x)$

$= -x^3 + x = -(x^3 - x)$

$= -f(x)$

When $f(-x) = -f(x) \Rightarrow$ It is an odd function

$\Rightarrow$ symmetric with respect to the origin.

Review even, odd, or neither functions

Mar 6-8:47 AM

4) Find $f'(x)$ using the definition of $f'(x)$.

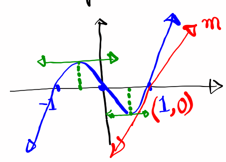
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - (x+h) - x^3 + x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{-x} - h \cancel{-x^3} \cancel{+x}}{h}$$

Review binomial Thrm $(x+y)^n = \ldots$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + \cancel{3xh}^{\to 0} + \cancel{h^2}^{\to 0} - 1)}{\cancel{h}} \quad \boxed{3x^2 - 1}$$

5) Find eqn of tan. line to the graph of $f(x)$ at $x=1$.




$m = f'(1) = 3(1)^2 - 1 = 3 - 1 = 2$

$y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 1)$

$\boxed{y = 2x - 2}$

6) Find x-values where tan. lines are horizontal.

$m = 0$

$f'(x) = 0$

$3x^2 - 1 = 0$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

$x = \pm \frac{1}{\sqrt{3}}$

$\boxed{x = \pm \frac{\sqrt{3}}{3}}$

Mar 6-8:52 AM

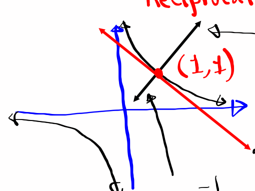
Find eqn of the normal line at $x=1$ to the graph of $\boxed{f(x) = \frac{1}{x}}$

$f(1) = 1$

Reciprocal Function

It is perpendicular to tan. line at tan. Point

From Algebra $m_1 \cdot m_2 = -1$ Perpendicular lines




$m = f'(1)$

$m = \frac{-1}{f'(1)}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{x - (x+h)}{h(x+h) \cdot x} = \lim_{h \to 0} \frac{-h}{h(x+h)x}$$

LCD $= (x+h) \cdot x$

$$= \lim_{h \to 0} \frac{-1}{(x+h) \cdot x} = \frac{-1}{x^2}$$

$f'(1) = -1$

$m_{\text{Normal Line}} = \frac{-1}{f'(1)} = \frac{-1}{-1} = 1$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 1) \rightarrow \boxed{y = x}$ Normal line at $(1,1)$

Mar 6-9:04 AM

Given $f(x) = \cos x$ Review $\cos(A+B)$

1) Find $f\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \cos 45^\circ = \frac{\sqrt{2}}{2}$

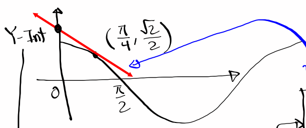
2) Find $f'(x)$

$$f'(x) = \lim_{h\to 0}\frac{\cos(x+h) - \cos x}{h} = \lim_{h\to 0}\frac{\cos x\cosh - \sin x\sin h - \cos x}{h}$$

$$= \lim_{h\to 0}\frac{\cos x(\cosh - 1)}{h} - \lim_{h\to 0}\frac{\sin x\sin h}{h}$$

$$= \cos x \cdot \lim_{h\to 0}\frac{\cosh - 1}{h} - \sin x \cdot \lim_{h\to 0}\frac{\sin h}{h}$$

$$= \cos x \cdot 0 - \sin x \cdot 1 = \boxed{-\sin x}$$




$m = f'\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$y - y_1 = m(x - x_1)$

$y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$\boxed{y = -\frac{\sqrt{2}}{2}x + \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}}$

$\left(0, \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}\right)$

Mar 6-9:13 AM

This is what we know so far,

$$\frac{d}{dx}[\sin x] = \cos x \quad \& \quad \frac{d}{dx}[\cos x] = -\sin x$$

Prove $\frac{d}{dx}[c] = 0$

Let $f(x) = c$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{c - c}{h}$$

$$= \lim_{h\to 0}\frac{0}{h} = \lim_{h\to 0} 0 = \boxed{0}$$

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For $n$ as positive integer,

Prove $\frac{d}{dx}[x^n] = n x^{n-1}$ Power Rule

$f(x) = x^n$ , $f(x+h) = (x+h)^n$

Binomial Theorem

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \cdots + h^n \cancel{- x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} h + \cdots + h^{n-1}]}{\cancel{h}}$$

$= \boxed{n x^{n-1}}$ Power Rule

ex: $f(x) = x^5$

$f'(x) = 5x^{5-1} = 5x^4$

ex: $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = 1/2\, x^{1/2 - 1}$ → $f'(x) = \frac{1}{2} x^{-1/2} = \boxed{\frac{1}{2\sqrt{x}}}$

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So far we know

$\frac{d}{dx}[\sin x] = \cos x$ , $\frac{d}{dx}[\cos x] = -\sin x$ , $\frac{d}{dx}[c] = 0$,

$\frac{d}{dx}[x^n] = n x^{n-1}$

Now more rules

$\frac{d}{dx}[c f(x)] = c \frac{d}{dx}[f(x)]$

$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

$\frac{d}{dx}[\frac{1}{4}x^4] = \boxed{\frac{1}{4}} \frac{d}{dx}[x^4] = \frac{1}{4} \cdot 4x^3 = \boxed{x^3}$

$\frac{d}{dx}[10 + \sin x] = \frac{d}{dx}[10] + \frac{d}{dx}[\sin x] = 0 + \cos x = \boxed{\cos x}$

$\frac{d}{dx}[x^2 - \cos x] = \frac{d}{dx}[x^2] - \frac{d}{dx}[\cos x]$

$= 2x^{2-1} - (-\sin x)$

$= \boxed{2x + \sin x}$

Mar 6-9:37 AM

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

Product Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

Quotient Rule

$$\frac{d}{dx}\left[x^2\cdot \cos x\right] = \frac{d}{dx}[x^2]\cdot \cos x + x^2\cdot\frac{d}{dx}[\cos x]$$

$$= 2x\cos x + x^2\cdot(-\sin x)$$

$$= 2x\cos x - x^2\sin x$$

$$\frac{d}{dx}\left[\frac{x}{x-1}\right] = \frac{\frac{d}{dx}[x]\cdot(x-1) - x\cdot\frac{d}{dx}[x-1]}{(x-1)^2}$$

$$= \frac{1(x-1) - x\cdot 1}{(x-1)^2} = \boxed{\frac{-1}{(x-1)^2}}$$

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