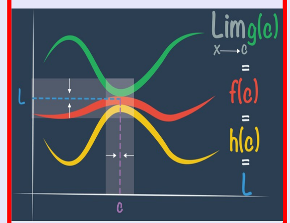
Calculus I

Lecture 16



Class QZ 8

Given $\boxed{f(x) = x^2 - 2x}$

Y-Int (0,0)

x-Int (0,0), (2,0)

1) Find $\boxed{f'(x)}$ using the $\boxed{\text{definition of } f'(x).}$

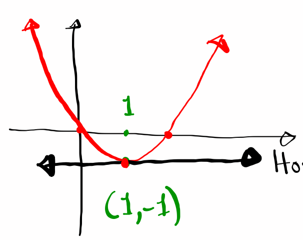
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - 2x - 2h \cancel{-x^2} + 2x}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + \overset{\to 0}{h} - 2)}{\cancel{h}} = \boxed{2x - 2}$$

2) Find x-value where $f'(x) = 0$.

$$2x - 2 = 0$$
$$2x = 2$$
$$\boxed{x = 1}$$

	x=0	1	x=2
$f'(x)$	−		+
$f(x)$	Dec.		Inc.



Horizontal tan. line at $x = 1$

$$m = f'(1) = 2(1) - 2 = \boxed{0}$$

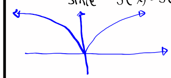
$$f(1) = 1^2 - 2(1) = \boxed{-1}$$

Given $f(x) = \sqrt[3]{x^2}$

1) Is $f(x)$ even, odd, or neither?

$$f(-x) = \sqrt[3]{(-x)^2} = \sqrt[3]{x^2} = f(x)$$

Since $f(-x) = f(x)$, $f(x)$ is an even function.

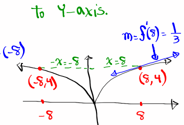


Even functions are symmetric with respect to Y-axis.

2) Find $f(8)$ and $f(-8)$

$$f(8) = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

$$f(-8) = \sqrt[3]{(-8)^2} = \sqrt[3]{64} = 4$$



$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$$

3) Find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt[3]{(x+h)^2} - \sqrt[3]{x^2}}{h} = \frac{0}{0} \text{ I.F.}$$

$$= \lim_{h \to 0} \frac{\overset{A}{\sqrt[3]{(x+h)^2}} - \overset{B}{\sqrt[3]{x^2}}}{h} \cdot \frac{\overset{A^2}{\left(\sqrt[3]{(x+h)^2}\right)^2} + \overset{A\ B}{\sqrt[3]{(x+h)^2} \cdot \sqrt[3]{x^2}} + \overset{B^2}{\left(\sqrt[3]{x^2}\right)^2}}{\left(\sqrt[3]{(x+h)^2}\right)^2 + \sqrt[3]{(x+h)^2} \cdot \sqrt[3]{x^2} + \left(\sqrt[3]{x^2}\right)^2}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt[\cancel{3}]{(x+h)^2}\right)^{\cancel{3}} - \left(\sqrt[\cancel{3}]{x^2}\right)^{\cancel{3}}}{h\left(\left(\sqrt[3]{(x+h)^2}\right)^2 + \sqrt[3]{(x+h)^2} \cdot \sqrt[3]{x^2} + \left(\sqrt[3]{x^2}\right)^2\right)}$$

$$\cancel{x^2} + 2xh + h^2 - \cancel{x^2}$$

$$2xh + h^2$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h\left[\sqrt[3]{(x+h)^4} + \sqrt[3]{(x+h)^2 \cdot x^2} + \sqrt[3]{x^4}\right]}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}\left[\sqrt[3]{(x+h)^4} + \sqrt[3]{(x+h)^2 x^2} + \sqrt[3]{x^4}\right]}$$

$$= \frac{2x}{\sqrt[3]{x^4} + \sqrt[3]{x^4} + \sqrt[3]{x^4}} = \frac{2x}{3\sqrt[3]{x^4}} = \frac{2\cancel{x}}{3\cancel{\sqrt[3]{x^3}}\sqrt[3]{x}}$$

$$= \boxed{\frac{2}{3\sqrt[3]{x}}}$$

$$f(x) = \sqrt[3]{x^2}$$

$$f'(x) = \frac{2}{3\sqrt[3]{x}}$$

$$f'(8) = \frac{2}{3\sqrt[3]{8}} = \frac{\cancel{2}}{3 \cdot \cancel{2}} = \boxed{\frac{1}{3}}$$

Find the first derivative of any linear function.

$$f(x) = mx + b$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{m(x+h) \cancel{+b} - mx \cancel{-b}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{mx} + mh - \cancel{mx}}{h} = \lim_{h \to 0} \frac{m\cancel{h}}{\cancel{h}} = \lim_{h \to 0} m = \boxed{m}$$

$f(x) = 2\sin x - x$

Find x values over $[0, 2\pi)$ where $\boxed{\text{tan. line is horizontal.}}$

Horizontal lines have $\underbrace{\text{zero slope}}_{f'(x)=0}$.

$f(x) = 2\sin x - x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2\sin(x+h) - (x+h) - 2\sin x + x}{h}$$

$$= \lim_{h \to 0} \left(\frac{2\sin(x+h) - 2\sin x}{h} - \frac{\cancel{x} + h - \cancel{x}}{h} \right)$$

$$= \lim_{h \to 0} \left[\frac{2(\sin(x+h) - \sin x)}{h} - 1 \right]$$

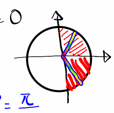
$$= 2 \boxed{\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}} - \lim_{h \to 0} 1$$

$$f'(x) = \boxed{2\cos x - 1}$$

we want $f'(x) = 0$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$



R.S. Angle $60° = \frac{\pi}{3}$

QI $\rightarrow x = \boxed{\frac{\pi}{3}}$

QIV $\rightarrow x = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$

Other notation for derivative

For $f(x) \rightarrow f'(x)$ First derivative

$$\frac{df}{dx} = \frac{d}{dx}[f(x)] \quad " \quad "$$

If $y = f(x) \rightarrow$ First derivative $\boxed{y' = \frac{dy}{dx}}$

f'' F-double Prime $\rightarrow$ Second derivative

$$y'' = \frac{d}{dx}[y'] = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d^2y}{dx^2}$$

Given $y = ax^2 + bx + C$

Find y' and y''.

$$y' = \lim_{h \to 0} \frac{\overbrace{\boxed{a(x+h)^2 + b(x+h) + C}}^{f(x+h)} \overbrace{\boxed{-ax^2 - bx - C}}^{-f(x)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{ax^2} + 2axh + ah^2 \cancel{+bx} + bh \cancel{+C} \cancel{-ax^2} \cancel{-bx} \cancel{-C}}{h}$$

$$= \lim_{h \to 0} \frac{2axh + ah^2 + bh}{h} = \lim_{h \to 0} \frac{\cancel{h}(2ax + \cancel{ah}^{\to 0} + b)}{\cancel{h}} \boxed{2ax+b}$$

Linear Function

$$y'' = \lim_{h \to 0} \frac{\overbrace{2a(x+h) \cancel{+b}}^{\overset{f(x+h)}{\cancel{2ax} + 2ah}} \overbrace{- \cancel{2ax} \cancel{-b}}^{-f(x)}}{h} = \lim_{h \to 0} \frac{2a\cancel{h}}{\cancel{h}} = \boxed{2a}$$

$y = ax^2 + bx + C$

$y' = 2ax + b$ $\Rightarrow \boxed{y''' = 0}$

$y'' = 2a$ ← Constant