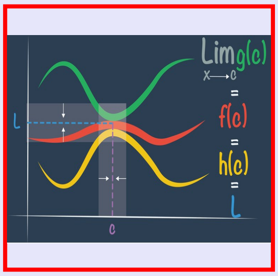
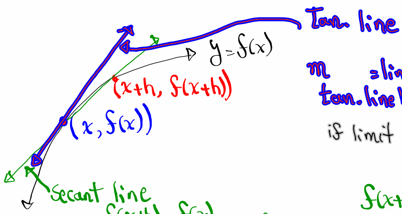
Calculus I

Lecture 15



Feb 19-8:47 AM



$$\underset{\text{tan. line}}{m} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if limit exists.

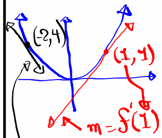
$$m = \frac{f(x+h) - f(x)}{x+h-x} \Rightarrow m_{\text{secant line}} = \frac{f(x+h) - f(x)}{h}$$

Definition : The function $f'(x)$ "f-prime of x"

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f'(x)$ is the slope of the tan. lines to graph of $f(x)$.

$f(x) = x^2$



$m = f'(1)$

$= 2(1) = \boxed{2}$

$m = f'(-2) = 2(-2) = \boxed{-4}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 2x + 0 = \boxed{2x}$$

Mar 4-8:47 AM

Given $f(x)=\frac{x}{x-2}$, find $f'(x)$

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{\frac{x+h}{x+h-2}-\frac{x}{x-2}}{h}$$

LCD $=(x+h-2)\cdot(x-2)$

$$=\lim_{h\to0}\frac{(x-2)(x+h)-(x+h-2)\cdot x}{(x+h-2)(x-2)\cdot h}=\lim_{h\to0}\frac{\cancel{x^2}\cancel{+xh}\cancel{-2x}-2h\cancel{-x^2}\cancel{-hx}\cancel{+2x}}{(x+h-2)(x-2)\cdot h}$$

$$=\lim_{h\to0}\frac{-2\cancel{h}}{(x+h-2)(x-2)\cdot\cancel{h}}=\frac{-2}{(x+0-2)(x-2)}=\boxed{\frac{-2}{(x-2)^2}}$$

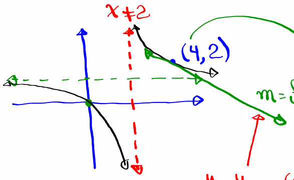
$f(x)=\frac{x}{x-2}$ $\quad$ $x\neq2$

$f'(x)=\frac{-2}{(x-2)^2}$ $\quad$ $x\neq2$

$f(4)=\frac{4}{4-2}$

$=\frac{4}{2}$

$=2$



$m=f'(4)=\frac{-2}{(4-2)^2}$

$=\frac{-2}{4}=\boxed{\frac{-1}{2}}$ slope of the tan. line at $x=4$

$y-y_1=m(x-x_1)$

$y-2=\frac{-1}{2}(x-4)$

$y-2=\frac{-1}{2}x+2$

$\boxed{y=\frac{-1}{2}x+4}$

Mar 4-8:58 AM

Given $f(x)=\sqrt{x}$

1) Find $f(4)=\sqrt{4}=\boxed{2}$ $\quad$ (x ↑, ↑ y)

2) Find $f'(x)=\boxed{\frac{1}{2\sqrt{x}}}$

3) Find $f'(4)=\frac{1}{2\sqrt{4}}=\boxed{\frac{1}{4}}$

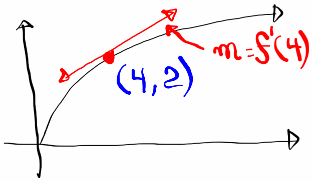
$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\lim_{h\to0}\frac{\cancel{x}\cancel{+h}^{\,1}\cancel{-x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$=\frac{1}{\sqrt{x}+\sqrt{x}}=\boxed{\frac{1}{2\sqrt{x}}}$$

4) Find $\boxed{\text{eqn of tan. line}}$ to the graph of $f(x)=\sqrt{x}$ at $x=4$.



$m=f'(4)=\frac{1}{4}$

$y-y_1=m(x-x_1)$

$y-2=\frac{1}{4}(x-4)$

$y=\frac{1}{4}x-1+2$

$\boxed{y=\frac{1}{4}x+1}$

Mar 4-9:11 AM

Given $f(x)=x^3$

1) Find $f(2)=2^3=8 \rightarrow (2,8)$

2) Find $f'(x)=\boxed{3x^2}$

3) Find $f'(2)=3(2)^2 = \boxed{12}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

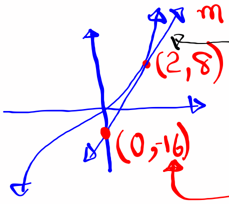
$$=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-x^3}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}=3x^2+3x(0)+0^2=\boxed{3x^2}$$

4) Find eqn of tan. line to the graph of $f(x)=x^3$ at $x=2$.

$m=f'(2)=12$



$y-y_1=m(x-x_1)$

$y-8=12(x-2)$

$y=12x-24+8$

$\boxed{y=12x-16}$

Mar 4-9:24 AM

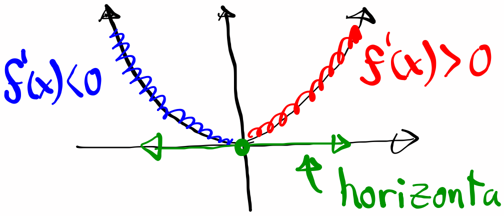
when $f'(x)>0 \rightarrow f(x)$ is increasing

when $f'(x)<0 \rightarrow f(x)$ is decreasing

when $f'(x)=0 \rightarrow f(x)$ has either max or Min. Point.

Tan. line is horizontal

$f(x)=x^2$, $f'(x)=2x$



horizontal tan. line $(m=0)$

Mar 4-9:36 AM

Class QZ 8

Given $f(x) = x^2 - 2x$

1) Find $f'(x)$ using the definition of $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - \cancel{x^2} + \cancel{2x}}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h-2)}{\cancel{h}} = \boxed{2x-2}$$

2) Find x-value where $f'(x) = 0$.

$$2x - 2 = 0$$

$$2x = 2 \rightarrow \boxed{x = 1}$$