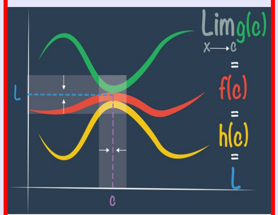
Calculus I

Lecture 13



Feb 19-8:47 AM

Two known limits

1) $\lim\limits_{h\to0}\dfrac{\sin h}{h}=1$

2) $\lim\limits_{h\to0}\dfrac{1-\cos h}{h}=0$

Prove $\lim\limits_{h\to0}\dfrac{1-\cos h}{h}=0$ ✓

By Direct plug in,

$\dfrac{1-\cancelto{1}{\cos 0}}{0}=\dfrac{0}{0}$ I.F.

$$\lim_{h\to0}\frac{1-\cos h}{h}\cdot\frac{1+\cos h}{1+\cos h}=\lim_{h\to0}\frac{1-\cos^2 h}{h(1+\cos h)}$$

$$=\lim_{h\to0}\frac{\sin^2 h}{h(1+\cos h)}=\lim_{h\to0}\left[\frac{\sin h}{h}\cdot\frac{1}{1+\cos h}\cdot\sin h\right]$$

$$=\cancelto{1}{\lim_{h\to0}\frac{\sin h}{h}}\cdot\lim_{h\to0}\frac{1}{1+\cos h}\cdot\lim_{h\to0}\sin h$$

$$=1\cdot\cancelto{\frac{1}{2}}{\frac{1}{1+\cos 0}}\cdot\cancelto{0}{\sin 0}=1\cdot\frac{1}{2}\cdot0=\boxed{0}$$

Feb 26-8:49 AM

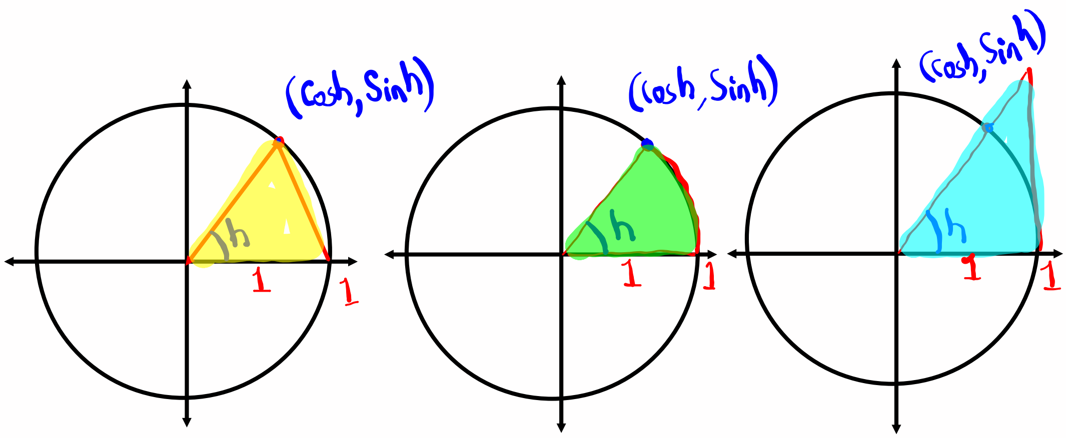
Find $\lim\limits_{h\to 0} \dfrac{h^2}{1-\cos h} = \dfrac{0^2}{1-\cos 0} = \dfrac{0}{0}$ I.F.

$$\lim_{h\to 0} \frac{h^2}{1-\cos h}\cdot\frac{1+\cos h}{1+\cos h} = \lim_{h\to 0}\frac{h^2(1+\cos h)}{\sin^2 h}$$

Divide by h^2

$$= \lim_{h\to 0}\frac{1+\cos h}{\frac{\sin^2 h}{h^2}} = \frac{1+1}{\left(\cancelto{1}{\lim\limits_{h\to 0}\frac{\sin h}{h}}\right)^2} = \frac{2}{1^2} = \boxed{2}$$

Feb 27-8:51 AM



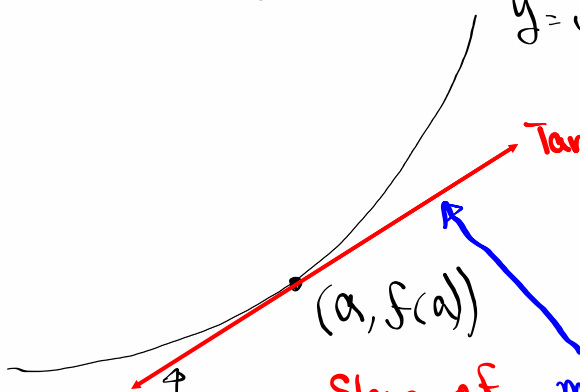
Yellow < Green < Blue

Triangle Sector Triangle

Feb 27-8:55 AM

Consider the graph below

$y = f(x)$



Tangent line at $(a, f(a))$

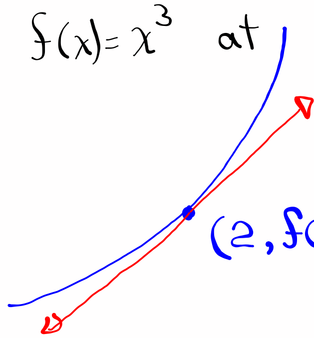
$(a, f(a))$

Slope of tangent line at $x = a$ $\quad m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

eqn of tan. line $\quad y - f(a) = m(x - a)$

Feb 27-9:01 AM

find eqn of the tangent line to the graph of $f(x) = x^3$ at $x = 2$.



$(2, f(2)) = (2, 8)$

$$m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^3 - 8}{x - 2}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{x-2}}$$

$$= \lim_{x \to 2} (x^2 + 2x + 4)$$

$$= \boxed{12}$$

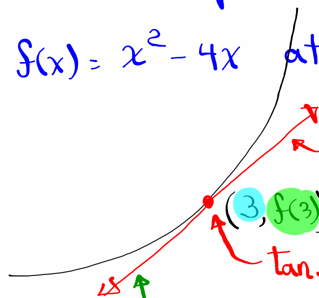
$y - y_1 = m(x - x_1)$

$y - 8 = 12(x - 2)$

$\boxed{y = 12x - 16}$

Feb 27-9:05 AM

Find the eqn of tan. line to the graph of $f(x) = x^2 - 4x$ at $x = 3$.



$m = \lim_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$

$(3, f(3)) = (3, -3)$

tan. Point

$= \lim_{x \to 3} \dfrac{x^2 - 4x - (-3)}{x - 3}$

$= \lim_{x \to 3} \dfrac{x^2 - 4x + 3}{x - 3}$

$= \lim_{x \to 3} \dfrac{\cancel{(x-3)}(x-1)}{\cancel{x-3}}$

$= \lim_{x \to 3} (x - 1)$

$= \boxed{2}$

slope of tan. line at $x=3$

$y - y_1 = m(x - x_1)$

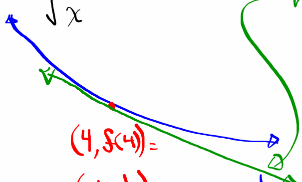
$y - (-3) = 2(x - 3)$

$y + 3 = 2x - 6$

$\boxed{y = 2x - 9}$

Feb 27-9:10 AM

Find eqn of tan. line to the graph of $f(x) = \dfrac{1}{\sqrt{x}}$ at $x = 4$.



$(4, f(4)) = (4, \frac{1}{2})$

$m = \frac{-1}{16}$

$m = \lim_{x \to 4} \dfrac{f(x) - f(4)}{x - 4}$

$= \lim_{x \to 4} \dfrac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x - 4}$

LCD: $2\sqrt{x}$

$= \lim_{x \to 4} \dfrac{2 - \sqrt{x}}{2\sqrt{x}(x-4)}$

$= \lim_{x \to 4} \dfrac{\boxed{(2 - \sqrt{x}) \cdot (2 + \sqrt{x})}}{2\sqrt{x}(x-4) \cdot (2 + \sqrt{x})} = \lim_{x \to 4} \dfrac{\overset{-1}{\cancel{4 - x}}}{2\sqrt{x}\cancel{(x-4)}(2 + \sqrt{x})}$

$= \lim_{x \to 4} \dfrac{-1}{2\sqrt{x}(2 + \sqrt{x})} = \dfrac{-1}{2\sqrt{4}(2 + \sqrt{4})} = \boxed{\dfrac{-1}{16}}$

$\dfrac{a-b}{b-a} = -1$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{-1}{16}(x - 4)$

$y = \frac{-1}{16}x + \frac{4}{16} + \frac{1}{2}$

$y = \frac{-1}{16}x + \frac{1}{4} + \frac{1}{2} \Rightarrow \boxed{y = \frac{-1}{16}x + \frac{3}{4}}$

Eqn. of tan. line

Feb 27-9:16 AM

Find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x)=\cos x$.

$$\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}=\lim_{h\to 0}\frac{\cos x\cosh-\sin x\sinh-\cos x}{h}$$

$$=\lim_{h\to 0}\frac{\cos x[\cosh-1]-\sin x\sinh}{h}$$

$$=\lim_{h\to 0}\left[\frac{\cos x(\cosh-1)}{h}-\frac{\sin x\sinh}{h}\right]$$

$$=\lim_{h\to 0}\frac{\cos x(\cosh-1)}{h}-\lim_{h\to 0}\frac{\sin x\sinh}{h}$$

$$=\cos x\cdot\underbrace{\lim_{h\to 0}\frac{\cosh-1}{h}}_{\to 0}-\sin x\cdot\underbrace{\lim_{h\to 0}\frac{\sinh}{h}}_{\to 1}$$

$$=\cos x\cdot 0-\sin x\cdot 1$$

$$=\boxed{-\sin x}$$

Feb 27-9:26 AM

Evaluate $\lim_{h\to 0}\frac{\sin^2 h}{h}=\frac{\sin^2 0}{0}=\frac{0}{0}$ I.F.

$x^2=x\cdot x$

$$\lim_{h\to 0}\frac{\sin^2 h}{h}=\lim_{h\to 0}\frac{\sinh\cdot\sinh}{h}$$

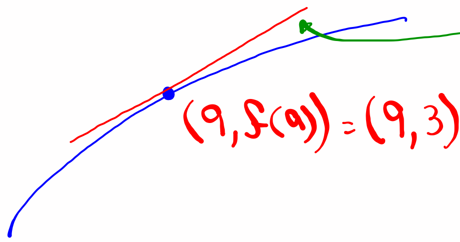
$$=\lim_{h\to 0}\sinh\cdot\underbrace{\lim_{h\to 0}\frac{\sinh}{h}}_{\to 1}$$

$$=\sin 0\cdot 1=0\cdot 1=\boxed{0}$$

Feb 27-9:34 AM

Class QZ 7

Find $\boxed{\text{slope of the tan. line}}$ to the graph of $f(x) = \sqrt{x}$ at $x = 9$.



$$m = \lim_{x \to 9} \frac{f(x) - f(9)}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

$$= \lim_{x \to 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{(x - 9)(\sqrt{x} + 3)} = \lim_{x \to 9} \frac{x - 9}{(x-9)(\sqrt{x} + 3)} = \lim_{x \to 9} \frac{1}{\sqrt{x} + 3} = \boxed{\frac{1}{6}}$$

Feb 27-9:41 AM