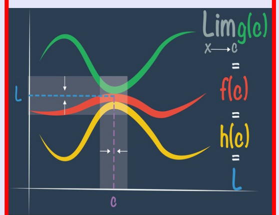
Calculus I

Lecture 12



Feb 19-8:47 AM

Two known limits

1) $\lim\limits_{h \to 0} \dfrac{\sin h}{h} = 1$

2) $\lim\limits_{h \to 0} \dfrac{1 - \cos h}{h} = 0$

1) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 5x}{x} = \lim\limits_{x \to 0} \dfrac{5 \sin 5x}{5x}$

$h = 5x$

$x \to 0,\ h \to 0$

$= 5 \lim\limits_{x \to 0} \dfrac{\sin 5x}{5x}$

$= 5 \lim\limits_{h \to 0} \dfrac{\sin h}{h} = 5 \cdot 1 = \boxed{5}$

2) $\lim\limits_{x \to 1} \dfrac{1 - \cos(x-1)}{x-1}$

$h = x - 1$

$x \to 1,\ h \to 0$

$= \lim\limits_{h \to 0} \dfrac{1 - \cos h}{h} = \boxed{0}$

Feb 26-8:49 AM

$$3)\ \lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{\frac{2\sin 2x}{2x}}{\frac{3\sin 3x}{3x}}$$

Let $x = .0001$

$$\frac{\sin .0002}{\sin .0003} \approx .6666667$$

Radian

$$= \frac{\lim\limits_{x\to 0} \frac{2\sin 2x}{2x}}{\lim\limits_{x\to 0} \frac{3\sin 3x}{3x}} = \frac{2\lim\limits_{x\to 0} \frac{\sin 2x}{2x}}{3\lim\limits_{x\to 0} \frac{\sin 3x}{3x}}$$

$\to 1$, $\to 1$

$$= \frac{2\cdot 1}{3\cdot 1} = \boxed{\frac{2}{3}} \approx .\overline{6}$$

Feb 26-8:55 AM

$$\lim_{x\to 2} \frac{1-\cos(x-2)}{x^2-4} = \lim_{x\to 2} \frac{1-\cos(x-2)}{(x+2)(x-2)}$$

$$= \boxed{\lim_{x\to 2} \frac{1}{x+2}} \cdot \lim_{x\to 2} \frac{1-\cos(x-2)}{x-2}$$

Let $x = 2.0001$

$$\frac{1-\cos(2.0001-2)}{2.0001^2-4} = 1.25\times 10^{-5} \approx .0000125$$

≈ 0

$h = x-2$

$x\to 2,\ h\to 0$

$$= \frac{1}{4}\cdot \boxed{\lim_{h\to 0} \frac{1-\cos h}{h}} \to 0$$

$$= \frac{1}{4}\cdot 0 = \boxed{0}$$

Feb 26-9:02 AM

Evaluate $\lim_{x \to 2} \frac{\sin(x-2)}{x^2-4} = \lim_{x \to 2} \frac{\sin(x-2)}{(x+2)(x-2)}$

Let $x = 2.0001$

$\frac{\sin(2.0001-2)}{2.0001^2-4} \approx .2499937497$

$\approx .25$

$= \boxed{\lim_{x \to 2} \frac{1}{x+2}} \cdot \lim_{x \to 2} \frac{\sin(x-2)}{x-2}$

$= \frac{1}{4} \cdot \boxed{\lim_{h \to 0} \frac{\sin h}{h}} \to 1$

$= \frac{1}{4} \cdot 1 = \boxed{\frac{1}{4}}$

Feb 26-9:08 AM

$f(x) = \sin x$, Evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) = \sin(x+h)$

$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

Recall from trig.

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$

$= \lim_{h \to 0} \frac{\sin x [\cos h - 1] + \cos x \sin h}{h} = \lim_{h \to 0} \frac{\sin x (\cos h - 1)}{h} + \lim_{h \to 0} \frac{\cos x \sin h}{h}$

$= \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} \to 0 + \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h} \to 1$

$= \sin x \cdot 0 + \cos x \cdot 1 = \boxed{\cos x}$

Make sure to review

$\cos(A+B) =$

Feb 26-9:13 AM

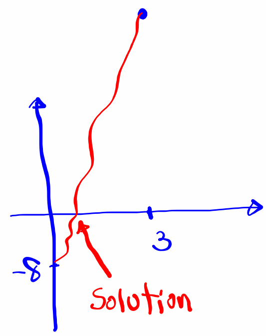
Show $\underbrace{x^4 - 2x^2 - 8}$ $= 0$ has a Solution on the interval $[0,3]$. $\rightarrow$ $f(x) = x^4 - 2x^2 - 8$

Polynomial Function

Continuous everywhere

$F(0) = -8$

$f(3) = +$



By I.V.T.

$f(x)$ is a cont. function on $[0,3]$

$f(0) < 0$, $f(3) > 0$

there is at least one number such that $f(x) = 0$

Feb 26-9:20 AM

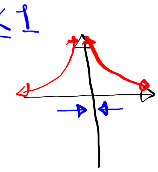
Evaluate $\lim_{x \to 0} \frac{\cos x}{x^2}$

Recall from Trig. $-1 \le \cos \alpha \le 1$

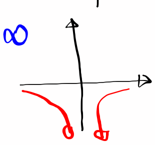
$-1 \le \cos x \le 1$

$\frac{-1}{x^2} \le \frac{\cos x}{x^2} \le \frac{1}{x^2}$

$\lim_{x \to 0} \frac{1}{x^2} = \infty$



$\lim_{x \to 0} \frac{-1}{x^2} = -\infty$



we can not use S.T.

$x \to 0^+ \rightarrow x = .001$

$\frac{\cos .001}{.001^2} = 999999.5$

$x \to 0^+$ $\frac{\cos x}{x^2} \to \infty$

$x \to 0^- \rightarrow x = -.001$

$\frac{\cos(-.001)}{(-.001)^2} = 999999.5$

$x \to 0^-$ $\frac{\cos x}{x^2} \to \infty$

$\lim_{x \to 0} \frac{\cos x}{x^2} = \infty$

Feb 26-9:26 AM

$$\lim_{x\to\infty}\left(\sqrt{x^2+4x}-x\right) = \infty - \infty \quad \text{I.F.}$$

$x = 1000$

$\sqrt{1000^2+4(1000)} - 1000$

$= 1.998 \approx 2$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+4x}-x}{1} \cdot \frac{\sqrt{x^2+4x}+x}{\sqrt{x^2+4x}+x}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}+4x-\cancel{x^2}}{\sqrt{x^2+4x}+x} = \frac{\infty}{\infty} \quad \text{I.F.}$$

$$= \lim_{x\to\infty} \frac{\frac{4x}{x}}{\frac{\sqrt{x^2+4x}}{x}+\frac{x}{x}} = \lim_{x\to\infty} \frac{4}{\sqrt{\frac{x^2+4x}{x^2}}+1}$$

$$= \lim_{x\to\infty} \frac{4}{\sqrt{1+\frac{4}{x}}+1} = \frac{4}{\sqrt{1}+1} = \frac{4}{2} = \boxed{2}$$

Feb 26-9:35 AM

$$\lim_{x\to-\infty} \frac{x}{\sqrt{9x^2-1}} = \frac{-\infty}{\infty} \quad \text{I.F.}$$

$x = -1000$

$$\frac{-1000}{\sqrt{9(-1000)^2-1}} =$$

$-.333333519 \approx \frac{-1}{3}$

$$\boxed{\begin{array}{l} x\to\infty \Rightarrow x = \sqrt{x^2} \\ x\to-\infty \Rightarrow x = -\sqrt{x^2} \end{array}}$$

$$\lim_{x\to-\infty} \frac{\frac{x}{x}}{\frac{\sqrt{9x^2-1}}{x}} = \lim_{x\to-\infty} \frac{1}{\frac{\sqrt{9x^2-1}}{-\sqrt{x^2}}}$$

when $x\to-\infty$

$x = -\sqrt{x^2}$

$$= \lim_{x\to-\infty} \frac{1}{-\sqrt{\frac{9x^2-1}{x^2}}}$$

$$= \lim_{x\to-\infty} \frac{1}{-\sqrt{9-\frac{1}{x^2}}}$$

$$= \frac{1}{-\sqrt{9}} = \boxed{\frac{-1}{3}}$$

Feb 26-9:42 AM

Prove $\lim_{x \to 5} (x^2 + 5x) = 50$

$f(x) = x^2 + 5x$

$a = 5$

$L = 50$

Verify $\lim_{x \to 5} (x^2+5x) = 50$ ✓

$5^2 + 5(5) = 25 + 25 = 50$ ✓

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 + 5x - 50| < \varepsilon$ 〃 $|x - 5| < \delta$

$|(x+10)(x-5)| < \varepsilon$ 〃 $|x - 5| < \delta$

$\underbrace{|x+10|}_{\text{Bound}}\underbrace{|x-5|}_{\text{Keep}} < \varepsilon$

Let $|x+10| < C \to C|x-5| < \varepsilon \to |x-5| < \frac{\varepsilon}{C}$

If we wish $\delta \le 1 \Rightarrow |x-5| < 1$

$-1 < x-5 < 1$

Add 15

$-1+15 < x-5+15 < 1+15$

$14 < x+10 < 16$

$|x+10| < \boxed{16}$ C

$\delta = \min\left\{1, \frac{\varepsilon}{16}\right\}$

$\boxed{\text{choose } \delta = \min\left\{1, \frac{\varepsilon}{16}\right\}}$

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