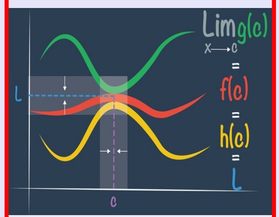
Calculus I

Lecture 11

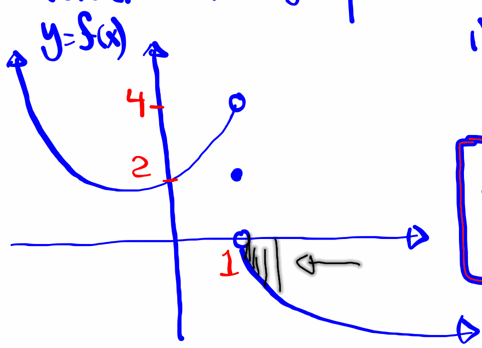


Feb 19-8:47 AM

Class QZ 6

$\boxed{\text{Box Your Final Ans.}}$

Consider the graph below



1) $\lim_{x \to 1^-} f(x) = \boxed{4}$

$\boxed{\text{2) } \lim_{x \to 1^+} f(x) = \boxed{0}}$ ✓

3) $\lim_{x \to 1} f(x) = \boxed{\text{D.N.E.}}$

4) $f(1) = \boxed{2}$

5) Is $f(x)$ cont. at $x=1$? $\boxed{\text{NO}}$

Feb 21-9:45 AM

For $\varepsilon = .05$, find a $\delta > 0$ such that $\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$.

$f(x) = \frac{x^2-4}{x-2}$

$a = 2$

$L = 4$

Verify $\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$ ✓

$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{0}{0}$ I.F.

$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x \to 2}(x+2) = 2+2 = 4$

For $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{x^2-4}{x-2} - 4\right| < .05$ whenever $|x-2| < \delta$

$|x + 2 - 4| < .05 \rightarrow |x-2| < .05$

$\boxed{\text{Choose } \delta = .05}$

Feb 22-8:46 AM

Prove $\lim_{x \to 0} \frac{x^2+x}{x} = 1$

$f(x) = \frac{x^2+x}{x}$

$a = 0$

$L = 1$

Verify $\lim_{x \to 0} \frac{x^2+x}{x} = 1$ ✓

$\lim_{x \to 0} \frac{x^2+x}{x} = \frac{0}{0}$ I.F.

$\lim_{x \to 0} \frac{x(x+1)}{x} = \lim_{x \to 0} (x+1) = 1$

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{x^2+x}{x} - 1\right| < \varepsilon$ whenever $|x - 0| < \delta$

$|x + \cancel{1} - \cancel{1}| < \varepsilon$ whenever $|x| < \delta$

$|x| < \varepsilon$ whenever $|x| < \delta$

$\boxed{\text{Choose } \delta = \varepsilon}$

Feb 22-8:53 AM

For $\varepsilon = .1$ Find $\delta > 0$ such that $\lim_{x \to 0} \sqrt[5]{x} = 0$

$f(x) = \sqrt[5]{x}$ verify $\lim_{x \to 0} \sqrt[5]{x} = 0$ ✓

$a = 0$

$L = 0$

For $\varepsilon = .1$, there is a $\delta > 0$ such that

$|f(x) - L| < .1$ whenever $|x - a| < \delta$

$|\sqrt[5]{x} - 0| < .1$ whenever $|x - 0| < \delta$

$|\sqrt[5]{x}| < .1$ " $|x| < \delta$

Raise both sides to 5th Power

$|\sqrt[5]{x}|^5 < .1^5$

$|x| < .1^5$

Pick (choose) $\delta = .1^5$

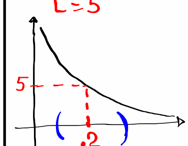
Feb 22-8:59 AM

For $\boxed{\varepsilon = .1}$ Find a $\delta > 0$ such that $\lim_{x \to 1/5} \frac{1}{x} = 5$

$f(x) = \frac{1}{x}$ $\lim_{x \to 1/5} \frac{1}{x} = \frac{1}{1/5} = 5$ ✓

$a = \frac{1}{5}$

$L = 5$



For $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{x} - 5\right| < \varepsilon$ " $\left|x - \frac{1}{5}\right| < \delta$

$\left|\frac{1 - 5x}{x}\right| < \varepsilon$

$\left|\frac{-5x + 1}{x}\right| < \varepsilon$

$\left|\frac{-5(x - \frac{1}{5})}{x}\right| < \varepsilon$

$\frac{|-5|}{|x|}\left|x - \frac{1}{5}\right| < \varepsilon$

$\frac{5}{|x|}\left|x - \frac{1}{5}\right| < \varepsilon$

If $\frac{5}{|x|} < C$

then $\left|x - \frac{1}{5}\right| < \frac{\varepsilon}{C}$

$\delta = \frac{\varepsilon}{C}$

Pick $\boxed{\delta \leq .1}$

$\left|x - \frac{1}{5}\right| \leq .1$

$|x - .2| \leq .1$

$-.1 \leq x - .2 \leq .1$

$.1 \leq x \leq .3$

$\frac{1}{10} \leq x \leq \frac{3}{10}$

$10 > \frac{1}{x} > \frac{10}{3}$

$-10 < \frac{10}{3} < \frac{1}{x} < 10$

$\frac{1}{|x|} < 10$

$\frac{5}{|x|} < 50$ ← C

$\delta = \min\left\{.1, \frac{\varepsilon}{50}\right\}$

For $\varepsilon = .1$

$\delta = \min\left\{.1, \frac{.1}{50}\right\}$

$\boxed{\delta = \frac{1}{500}}$

Feb 22-9:06 AM

Squeeze Thrm, Sandwich thrm, Pinching Thrm

If $g(x) \le f(x) \le h(x)$ for all points in an open interval where a is belong to and

$$\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$$

then $\lim_{x \to a} f(x) = L$ by Squeeze thrm

Evaluate $\lim_{x \to 0} x^4 \sin\frac{1}{x}$

Direct plug in $\to 0^4 \cdot \sin\left(\frac{1}{0}\right)$ undefined

$= 0 \cdot$

Recall from Trig.

$-1 \le \sin\alpha \le 1 \Rightarrow -1 \le \sin\frac{1}{x} \le 1$

multiply by x^4

$-x^4 \le x^4 \sin\frac{1}{x} \le x^4$

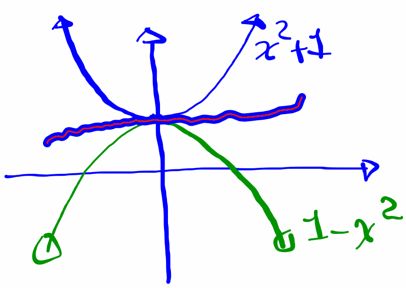
Now $\lim_{x \to 0} x^4 = 0$

$\lim_{x \to 0} -x^4 = 0$

by S.T., $\lim_{x \to 0} x^4 \sin\frac{1}{x} = 0$

Feb 22-9:21 AM

Suppose $1 - x^4 \le f(x) \le x^2 + 1$ for all x-values in the interval $(-1, 1)$. Find $\lim_{x \to 0} f(x)$.



$\lim_{x \to 0} (x^2 + 1) = 1$

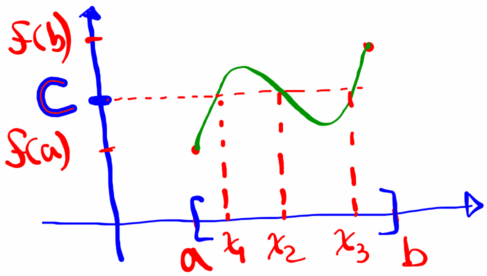
$\lim_{x \to 0} (1 - x^4) = 1$

By S.T., $\lim_{x \to 0} f(x) = 1$

Feb 22-9:29 AM

Intermediate - Value Theorem.

If $f(x)$ is a continuous function on $[a,b]$ and C is a number between $f(a)$ & $f(b)$,



$f(x_1)=C$

$f(x_2)=C$

$f(x_3)=C$

then there is at least one number x in $[a,b]$ such that $f(x)=C$.

Feb 22-9:34 AM

Show $\underbrace{x^3-4x+1}=0$ has a solution on $[1,2]$.

Degree

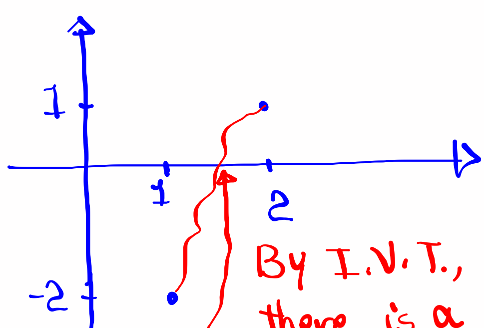
Let $f(x)=x^3-4x+1$

Polynomial function, Continuous everywhere

Not linear function

$f(1)=1^3-4(1)+1=-2$

$f(2)=2^3-4(2)+1=1$



By I.V.T., there is a number x such that $f(x)=0$

Feb 22-9:40 AM

Does $x^3 + x^2 = 2x + 1$ have a Solution in $[-1, 1]$?

at most 3 Solutions

$$\underbrace{x^3 + x^2 - 2x - 1}_{} = 0$$

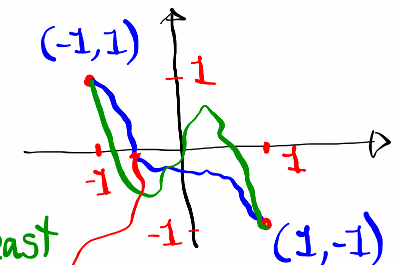
$$f(x) = x^3 + x^2 - 2x - 1$$

Polynomial

Continuous $(-\infty, \infty)$

$$f(-1) = (-1)^3 + (-1)^2 - 2(-1) - 1 = 1$$

$$f(1) = 1^3 + 1^2 - 2(1) - 1 = -1$$



By I.V.T., there is at least one number x in $[-1, 1]$ such that $f(x) = 0$

Feb 22-9:46 AM