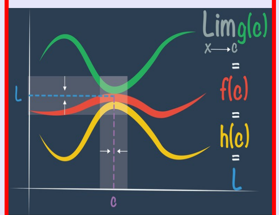
Calculus I

Lecture 10



Feb 19-8:47 AM

Prove $\lim_{x\to 3} x^2 = 9$

$f(x) = x^2$
$a = 3$
$L = 9$

Verify $\lim_{x\to 3} x^2 = 9$

$3^2 = 9$ ✓

For every $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 - 9| < \varepsilon$; $|x - 3| < \delta$

$|(x+3)(x-3)| < \varepsilon$; $|x - 3| < \delta$

$|ab| = |a||b|$

$|x+3|\,|x-3| < \varepsilon$

Isolate

Suppose $|x+3| < C$

$C|x-3| < \varepsilon$ → $|x - 3| < \frac{\varepsilon}{C}$

$\delta = \frac{\varepsilon}{C}$

If we wish $\delta \le 1$, then

$|x-3| < 1$

$-1 < x-3 < 1$ ← Add 3

$2 < x < 4$

$2+3 < x+3 < 4+3$

$5 < x+3 < 7$

$-7 < 5 < x+3 < 7$

$|x+3| < 7$

C

So $\delta = \min\left\{1, \frac{\varepsilon}{7}\right\}$

If $\varepsilon = 1$ → $\delta = \min\left\{1, \frac{1}{7}\right\} = \frac{1}{7}$

If $\varepsilon = 8$ → $\delta = \min\left\{1, \frac{8}{7}\right\} = 1$

If $\varepsilon = 7$ → $\delta = \min\left\{1, \frac{7}{7}\right\} = 1$

Feb 20-9:49 AM

Prove $\lim_{x \to 3} (x^2 + 2x - 4) = 11$

$f(x) = x^2 + 2x - 4$

$a = 3$

$L = 11$

Verify $\lim_{x \to 3} (x^2 + 2x - 4) = 11$ ✓

$3^2 + 2(3) - 4 = 9 + 6 - 4 = 11$

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 + 2x - 4 - 11| < \varepsilon$ whenever $|x - 3| < \delta$

$|x^2 + 2x - 15| < \varepsilon$ 〃 $|x - 3| < \delta$

$|(x+5)(x-3)| < \varepsilon$

$|x+5|\,|x-3| < \varepsilon$

Bound   Isolate

Suppose $|x+5| < C$

$C|x-3| < \varepsilon \to |x-3| < \frac{\varepsilon}{C}$

$\delta = \frac{\varepsilon}{C}$

If we wish $\boxed{\delta \le 1}$, then

$|x-3| < 1$

$-1 < x - 3 < 1$

Add 8

$-1+8 < x-3+8 < 1+8$

$7 < x+5 < 9$

$-9 < x+5 < 9$

$|x+5| < 9$

C

$\delta = \min\left\{1, \frac{\varepsilon}{9}\right\}$

If $\varepsilon = 1 \to \delta = \frac{1}{9}$

If $\varepsilon = 5 \to \delta = \frac{5}{9}$

If $\varepsilon = 18 \to \delta = \min\left\{1, \frac{18}{9}\right\} = \min\{1, 2\} = 1$

Feb 21-8:55 AM

Prove $\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$

$f(x) = \frac{1}{x}$ ← Not a Polynomial Function

$a = \frac{1}{2}$

$L = 2$

Verify $\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$

$\frac{1}{1/2} = 2$ ✓

For $\varepsilon > 0$, there is a $\delta > 0$ such that

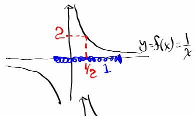
$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{x} - 2\right| < \varepsilon$ 〃 $\left|x - \frac{1}{2}\right| < \delta$

Some algebra

$$\left|\frac{1}{x} - 2\right| = \left|\frac{2}{x}\left(\frac{1}{2} - x\right)\right| = \left|\frac{-2}{x}\left(x - \frac{1}{2}\right)\right| = \left|\frac{-2}{x}\right|\left|x - \frac{1}{2}\right| = \frac{2}{|x|}\left|x - \frac{1}{2}\right|$$

If $\frac{2}{|x|} < C$, then $C\left|x - \frac{1}{2}\right| < \varepsilon$, $\left|x - \frac{1}{2}\right| < \frac{\varepsilon}{C}$ ← δ

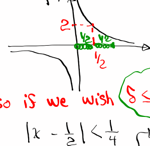


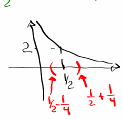
If we let $\delta \le 1$

$\frac{1}{2} - 1 = -\frac{1}{2}$ but $x \neq 0$

what about $\delta \le \frac{1}{2}$

$\frac{1}{2} - \frac{1}{2} = 0$ but $x \neq 0$





So if we wish $\delta \le \frac{1}{4}$

$\left|x - \frac{1}{2}\right| < \frac{1}{4}$

$-\frac{1}{4} < x - \frac{1}{2} < \frac{1}{4}$

$-\frac{1}{4} + \frac{1}{2} < x < \frac{1}{4} + \frac{1}{2} \to \frac{1}{4} < x < \frac{3}{4}$

$4 > \frac{1}{x} > \frac{4}{3}$

$8 > \frac{2}{x} > \frac{8}{3} \to \frac{8}{3} < \frac{2}{x} < 8$

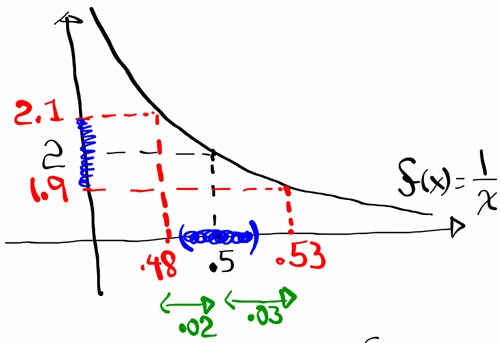
$\frac{2}{|x|} < 8$ ← C

So $\boxed{\delta = \min\left\{\frac{1}{4}, \frac{\varepsilon}{8}\right\}}$

If $\varepsilon = 1 \to \delta = \frac{1}{8}$

If $\varepsilon = .1 \to \delta = \frac{1}{80}$

Feb 21-9:08 AM



Suppose $\varepsilon = .1$

Solve $f(x) = 2.1$

$\frac{1}{x} = 2.1 \quad x = \frac{1}{2.1}$

$\boxed{x = .48}$

Solve $f(x) = 1.9$

$\frac{1}{x} = 1.9 \quad x = \frac{1}{1.9}$

$x = .53$

Pick $\delta = \min\{.02, .03\}$

$= .02$

$\delta = \min\left\{1, \frac{\varepsilon}{8}\right\}$

$\varepsilon = .1 \rightarrow \delta = \min\left\{1, \frac{.1}{8}\right\} = \min\left\{1, \frac{1}{80}\right\} = \frac{1}{80} = .0125 < .02$

Feb 21-9:29 AM

Prove $\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$

$f(x) = \frac{1}{x}$ ← Not a Polynomial Function

$a = \frac{1}{2}$

$L = 2$

Verify $\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$

$\frac{1}{1/2} = 2$ ✓

For $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{x} - 2\right| < \varepsilon$ " $\left|x - \frac{1}{2}\right| < \delta$

$\left|\frac{1 - 2x}{x}\right| < \varepsilon$

$\left|\frac{-2x + 1}{x}\right| < \varepsilon$

$\left|\frac{-2\left(x - \frac{1}{2}\right)}{x}\right| < \varepsilon$

$\frac{|-2|\left|x - \frac{1}{2}\right|}{|x|} < \varepsilon$

$\frac{2}{|x|}\left|x - \frac{1}{2}\right| < \varepsilon$

$\delta = \frac{\varepsilon}{C}$

See earlier work

$\delta = \min\left\{\frac{1}{4}, \frac{\varepsilon}{8}\right\}$

If $\frac{2}{|x|} < C$, then $\left|x - \frac{1}{2}\right| < \frac{\varepsilon}{C}$

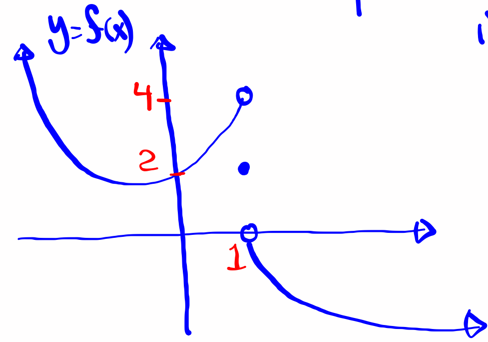
Google Squeeze thrm.

Feb 21-9:08 AM

Class QZ 6

$\boxed{\text{Box Your Final Ans.}}$

Consider the graph below



1) $\lim_{x \to 1^-} f(x) = \boxed{4}$

2) $\lim_{x \to 1^+} f(x) = \boxed{0}$

3) $\lim_{x \to 1} f(x) = \boxed{D.N.E.}$

4) $f(1) = \boxed{2}$

5) Is $f(x)$ cont. at $x=1$?

$\boxed{NO}$

Feb 21-9:45 AM