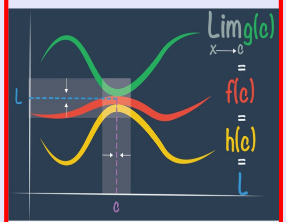
Math 261

Spring 2022

Lecture 9



Class QZ 5

1) Find $\lim\limits_{x\to\infty}\dfrac{2x-5}{3x+2} = \lim\limits_{x\to\infty}\dfrac{\frac{2x-5}{x}}{\frac{3x+2}{x}} = \lim\limits_{x\to\infty}\dfrac{2-\cancel{\frac{5}{x}}\to 0}{3+\cancel{\frac{2}{x}}\to 0}$

$= \boxed{\dfrac{2}{3}}$ ✓

2) Find $\lim\limits_{x\to-\infty}\dfrac{2x-5}{\sqrt{9x^2+2}} = \lim\limits_{x\to-\infty}\dfrac{\frac{2x-5}{x}}{\frac{\sqrt{9x^2+2}}{-\sqrt{x^2}}}$

as $x\to-\infty$

$x=-\sqrt{x^2}$

$= \lim\limits_{x\to-\infty}\dfrac{2-\frac{5}{x}}{-\sqrt{\frac{9x^2+2}{x^2}}} = -\lim\limits_{x\to-\infty}\dfrac{2-\cancel{\frac{5}{x}}\to 0}{\sqrt{9+\cancel{\frac{2}{x^2}}\to 0}} = \boxed{\dfrac{-2}{3}}$ ✓

Class QZ 6

Find $f'(x)$

1) $f(x) = x^4 \sin x$

Product Rule

$$\boxed{f'(x) = 4x^3 \cdot \sin x + x^4 \cdot \cos x}$$ ✓

$$\boxed{f'(x) = x^3 (4 \sin x + x \cos x)}$$ ✓

2) $f(x) = \dfrac{x}{x^2+1}$

Quotient Rule

$$f'(x) = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \boxed{\frac{1-x^2}{(x^2+1)^2}}$$ ✓

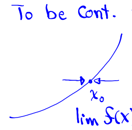
✓ $$f'(x) = \frac{-x^2+1}{(x^2+1)^2}$$

$$\cancel{f'(x) = -\frac{x^2+1}{(x^2+1)^2}}$$ Not Correct

$$f'(x) = -\frac{x^2-1}{(x^2+1)^2}$$ ✓

If $f(x)$ is differentiable at x_0, then $f(x)$ is continuous at x_0.

To be cont. at $x_0 \Rightarrow \lim_{h \to 0} f(x_0+h) = f(x_0)$



$\lim_{x \to x_0} f(x) = f(x_0)$

$$\lim_{h \to 0} [f(x_0+h) - f(x_0)] = 0$$

we need to prove this.

To be differentiable at x_0

$f'(x_0)$ exists $\Rightarrow \lim_{h \to 0} \dfrac{f(x_0+h) - f(x_0)}{h}$ exists.

So if $f(x)$ is differentiable at x_0, then

$$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = f'(x_0)$$

$$\lim_{h \to 0} [f(x_0+h) - f(x_0)] = \lim_{h \to 0} \left[\frac{f(x_0+h) - f(x_0)}{h} \cdot h \right]$$

$$\boxed{\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}} \cdot \boxed{\lim_{h \to 0} h}$$

$$= f'(x_0) \cdot 0$$

$$\lim_{h \to 0} [f(x_0+h) - f(x_0)] = 0$$

$$\lim_{h \to 0} f(x_0+h) - \lim_{h \to 0} f(x_0) = 0$$

$$\lim_{h \to 0} f(x_0+h) - f(x_0) = 0 \quad \Rightarrow \lim_{h \to 0} f(x_0+h) = f(x_0)$$

Continuity at x_0.

Function $f(x)$

First Derivative $f'(x) = \frac{d}{dx}[f(x)]$

Second Derivative $f''(x) = \frac{d^2}{dx^2}[f(x)]$

Third Derivative $f'''(x) = \frac{d^3}{dx^3}[f(x)]$

nth Derivative $f^{(n)}(x) = \frac{d^n}{dx^n}[f(x)]$

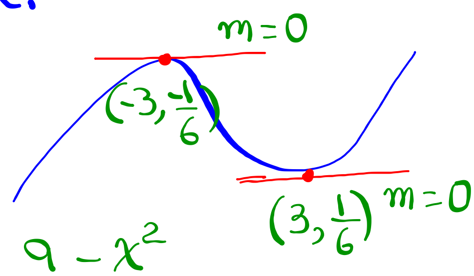
Ex.: $y = x^2 - 5x + 8$

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^2 - 5x + 8\right] = 2x - 5$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[2x - 5\right] = 2$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right] = \frac{d}{dx}\left[2\right] = 0$$

Find all points on $y = \dfrac{x}{x^2+9}$ that have horizontal tangent line.



$m = 0$

$f'(x) = 0$

$$y' = \frac{1(x^2+9) - x(2x)}{(x^2+9)^2} = \frac{9 - x^2}{(x^2+9)^2}$$

$$y' = 0 \rightarrow 9 - x^2 = 0 \rightarrow x^2 = 9 \rightarrow x = \pm 3$$

$x = 3 \Rightarrow y = \dfrac{3}{3^2+9} = \dfrac{3}{18} = \dfrac{1}{6}$ $\quad x = -3 \rightarrow y = \dfrac{-1}{6}$