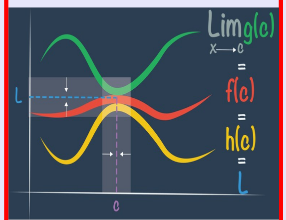
Math 261

Spring 2022

Lecture 21



$f(x) = x^2 \qquad f'(x) = 2x$

$f(x) = \sqrt{x} = x^{\frac{1}{2}} \qquad f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\left[x^n\right] = n\,x^{n-1}$

Reverse

$f'(x) = 2x^{\boxed{1}}$

$f(x) = 2\cdot\frac{x^{\boxed{1}+1}}{\boxed{1}+1} + C$

$= 2\frac{x^2}{2} + C$

$f(x) = x^2 + C$

Reverse of Power rule in differentiation

$\frac{x^{n+1}}{n+1} + C$

$f'(x) = 5x^3 \qquad f(x) = \frac{5x^{3+1}}{3+1} + C$

$\boxed{f(x) = \frac{5}{4}x^4 + C}$

Find $f(x)$

1) $f'(x) = 2\sqrt{x}$

$f'(x) = 2x^{.5}$

$f'(x) = 2x^{\frac{1}{2}}$ → Doing the reverse

$$f(x) = 2 \cdot \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= 2 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= 2 \cdot \frac{2}{3} x^{\frac{3}{2}} + C$$

$$\boxed{f(x) = \frac{4}{3} x\sqrt{x} + C}$$

$f(x) = 3x^2 + \cos x$ $\qquad$ $f'(x) = 6x - \sin x$

Suppose

$$f'(x) = \boxed{6x}\boxed{-\sin x}$$

$$f(x) = 6 \cdot \frac{x^{1+1}}{1+1} + \cos x + C$$

$$f(x) = \frac{6}{2} x^2 + \cos x + C$$

$$\boxed{f(x) = 3x^2 + \cos x + C}$$

Suppose $f'(x) = 4x^3 + \sec^2 x$ and $\boxed{f(0)=2}$

Find $f(x)$.

$$f(x) = \frac{\cancel{4}x^{3+1}}{\cancel{3+1}} + \tan x + C$$

$$f(x) = x^4 + \tan x + C$$

$$f(0) = \cancel{0^4}^{\,0} + \cancel{\tan 0}^{\,0} + C = 2$$

$$\boxed{C=2}$$

$$\boxed{f(x) = x^4 + \tan x + 2}$$

$f'(x) = 5x^4 - 4x^3 + 8$, $f(1) = 4$

Find $f(x)$.

$$f(x) = \frac{\cancel{5}x^{4+1}}{\cancel{4+1}} - \frac{\cancel{4}x^{3+1}}{\cancel{3+1}} + 8x + C$$

$$f(x) = x^5 - x^4 + 8x + C$$

$$f(1) = 1^5 - 1^4 + 8(1) + C = 4$$

$$\boxed{C=-4}$$

$$\boxed{f(x) = x^5 - x^4 + 8x - 4}$$

find $f(x)$

1) $f'(x) = \sqrt{x^3} + \sqrt[3]{x^2}$

$$f'(x) = x^{\frac{3}{2}} + x^{\frac{2}{3}}$$

$$f(x) = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C$$

$$f(x) = \frac{2}{5}\sqrt{x^5} + \frac{3}{5}\sqrt[3]{x^5} + C$$

2) $f'(x) = 2x - 3\sin x$, $f(0) = 5$

$$f(x) = x^2 + 3\cos x + C$$

$$f(0) = 0^{2} \to 0 + 3\cos 0 \to 1 + C = 5$$

$$\boxed{C = 2}$$

$$\boxed{f(x) = x^2 + 3\cos x + 2}$$

$f''(x) = 48x^2 - 6x + 1$ $\qquad f(0) = 1$

$\boxed{f'(0) = 2}$

$f'(x) = 48 \cdot \frac{x^3}{3} - 6 \cdot \frac{x^2}{2} + x + C$

$f'(x) = 16x^3 - 3x^2 + x + C$

$f'(0) = 16 \cdot 0^3 - 3 \cdot 0^2 + 0 + C = 2$

$C = 2$

$f'(x) = 16x^3 - 3x^2 + x^{\boxed{1}} + 2$

$f(x) = 16 \cdot \frac{x^4}{4} - 3 \cdot \frac{x^3}{3} + \frac{x^2}{2} + 2x + C$

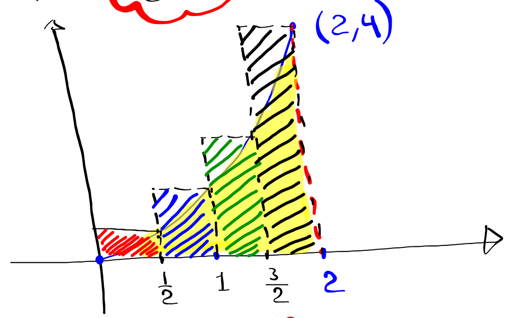
$= 4x^4 - x^3 + \frac{1}{2}x^2 + 2x + C$

$f(0) = 0 - 0 + 0 + 0 + C = 1$

$\boxed{C = 1}$

$$\boxed{f(x) = 4x^4 - x^3 + \frac{1}{2}x^2 + 2x + 1}$$

Graph $y = x^2$ from $x = 0$ to $x = 2$.

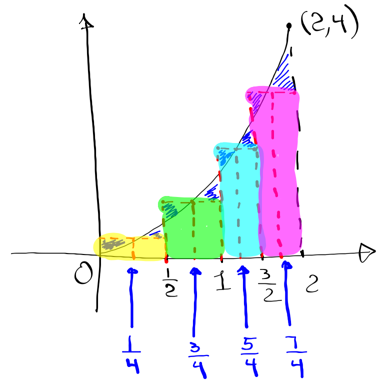


$R_1 \to A_1 = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{4}$

$R_2 \to A_2 = \frac{1}{2} \cdot \left(\frac{2}{2}\right)^2 = \frac{1}{2} \cdot \frac{4}{4}$

$R_3 \to A_3 = \frac{1}{2} \cdot \left(\frac{3}{2}\right)^2 = \frac{1}{2} \cdot \frac{9}{4}$

$R_4 \to A_4 = \frac{1}{2} \cdot \left(\frac{4}{2}\right)^2 = \frac{1}{2} \cdot \frac{16}{4}$



$y = x^2$

desired Area

$A = A_1 + A_2 + A_3 + A_4$

As # of rectangles increase

$\to$ we get a closer Answer for the area

why not having # of rectangles approach ∞

$R_1 \to A_1 = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^2$

$R_2 \to A_2 = \frac{1}{2} \cdot \left(\frac{3}{4}\right)^2$

$R_3 \to A_3 = \frac{1}{2} \cdot \left(\frac{5}{4}\right)^2$

$R_4 \to A_4 = \frac{1}{2} \cdot \left(\frac{7}{4}\right)^2$