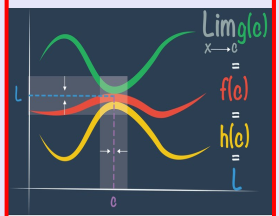
Math 261

Fall 2023

Lecture 9



Feb 19-8:47 AM

Evaluate $\lim\limits_{x\to 1} \dfrac{x^4+x-2}{x-1} = \dfrac{1^4+1-2}{1-1} = \dfrac{1+1-2}{1-1} = \dfrac{0}{0}$ I.F.

$\Rightarrow \lim\limits_{x\to 1} \dfrac{x^4+x-2}{x-1} = \lim\limits_{x\to 1} \dfrac{(x^3+x^2+x+2)\cancel{(x-1)}}{\cancel{x-1}}$

Num. $1^4+1-2=0$

Synthetic Div.

$$\begin{array}{c|ccccc} 1 & 1 & 0 & 0 & 1 & -2 \\ & & 1 & 1 & 1 & 2 \\ \hline & 1 & 1 & 1 & 2 & 0 \end{array}$$

$x-1$

other factor
x^3+x^2+x+2

$= \lim\limits_{x\to 1} (x^3+x^2+x+2)$

$= 1^3+1^2+1+2 = \boxed{5}$

Sep 12-10:25 AM

Given $f(x) = 3x^2 - 4x + 1$

Evaluate $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$$= \lim_{x \to a} \frac{3x^2 - 4x + 1 - (3a^2 - 4a + 1)}{x - a}$$

$$= \lim_{x \to a} \frac{3x^2 - 4x - 3a^2 + 4a}{x - a}$$

$$= \lim_{x \to a} \frac{3(x^2 - a^2) - 4(x - a)}{x - a}$$

$$= \lim_{x \to a} \frac{3(x+a)(x-a) - 4(x-a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}\left[3(x+a) - 4\right]}{\cancel{x-a}}$$

$$= \lim_{x \to a} \left[3(x+a) - 4\right] = 3(a+a) - 4$$
$$= \boxed{6a - 4}$$

Sep 12-10:31 AM

Given $f(x) = x^{-2}$

Evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{(x+h)^{-2} - x^{-2}}{h}$$

By Direct Subs. $\Rightarrow \frac{0}{0}$ I.F.

$$= \lim_{h \to 0} \frac{\left[(x+h)^{-1}\right]^2 - \left[x^{-1}\right]^2}{h} = \lim_{h \to 0} \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)\left(\frac{1}{x+h} + \frac{1}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2 \cdot x^2}$$

LCD $= (x+h)^2 \cdot x^2$

$$= \lim_{h \to 0} \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{h x^2 (x+h)^2} = \lim_{h \to 0} \frac{\cancel{h}(-2x - h)}{\cancel{h} x^2 (x+h)^2}$$

$$= \frac{-2x - 0}{x^2(x+0)^2} = \frac{-2x}{x^4} = \boxed{\frac{-2}{x^3}}$$

Sep 12-10:36 AM

Function $f(x)$ is $\boxed{\text{Continuous}}$ at $\boxed{x=a}$

if
1) $f(a)$ exists,
2) $\lim\limits_{x \to a} f(x)$ exists, and
3) $\lim\limits_{x \to a} f(x) = f(a)$

In terms of graphs, No hole, No gap, No jump at $x=a$.

Is $f(x) = \sqrt{x} + x$ cont. at $x=4$?

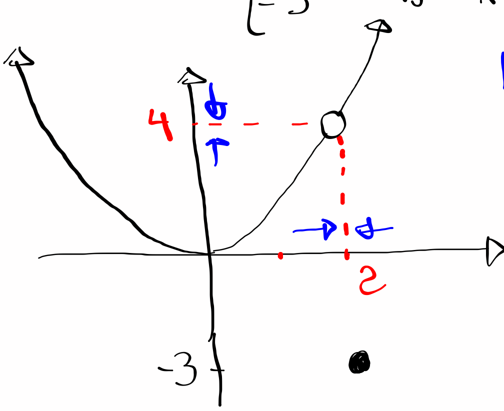
$f(4) = \sqrt{4} + 4 = 6$

$\lim\limits_{x \to 4} f(x) = \lim\limits_{x \to 4} [\sqrt{x} + x] = \sqrt{4} + 4 = 6$

Since $\lim\limits_{x \to 4} f(x) = f(4)$, then $f(x)$ is Cont. at $x=4$.

Sep 12-10:43 AM

Is $f(x) = \begin{cases} x^2 & \text{if } x \neq 2 \\ -3 & \text{if } x = 2 \end{cases}$ continuous at $x=2$?



$\lim\limits_{x \to 2} f(x) = 4$

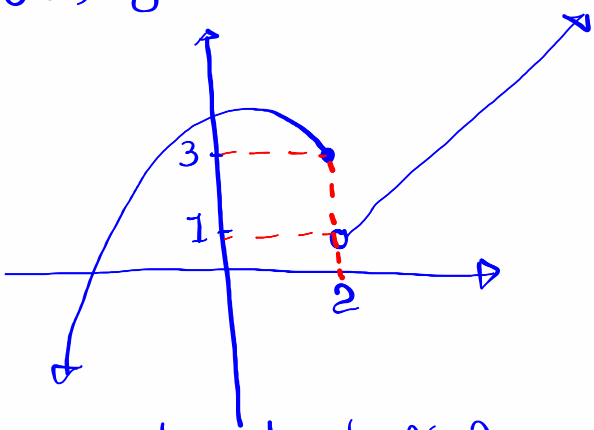
$F(2) = -3$

$\lim\limits_{x \to 2} f(x) \neq f(2)$

$\therefore f(x)$ is not Cont. at $x=2$.

Sep 12-10:48 AM

Consider the graph of $f(x)$ given below



Not cont. at $x=2$

$f(2)=3$

$\lim_{x\to 2^+} f(x)=1$

$\lim_{x\to 2^-} f(x)=3$

$\lim_{x\to 2} f(x)$ D.N.E.

Sep 12-10:51 AM

Use ε and δ definition to prove $\lim_{x\to 2}(5x-3)=7$

$f(x)=5x-3$

$L=7$

$a=2$

1) verify the limit

$\lim_{x\to 2}(5x-3)=5(2)-3=10-3=7$ ✓

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|5x-3-7|<\varepsilon$ whenever $|x-2|<\boxed{\delta}$

$|5x-10|<\varepsilon$

$|5(x-2)|<\varepsilon$

$|5|\,|x-2|<\varepsilon$

$5|x-2|<\varepsilon$

Divide by 5

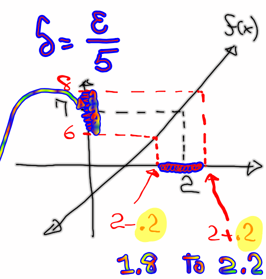
$|x-2|<\boxed{\frac{\varepsilon}{5}}$

$\delta=\frac{\varepsilon}{5}$

If $\varepsilon=1$, $\delta=\frac{1}{5}=.2$

choose $x=2.1$

$f(2.1)=5(2.1)-3=\boxed{7.5}$



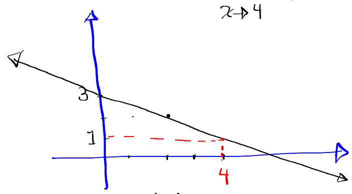
Sep 12-10:55 AM

Use ε and δ definition To prove $\lim_{x \to 4}\left(3-\frac{1}{2}x\right)=1$

$f(x)=3-\frac{1}{2}x$

$L=1$

$a=4$



Verify the limit $\quad \lim_{x \to 4}\left(3-\frac{1}{2}x\right)=3-\frac{1}{2}(4)=3-2=1$ ✓

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$\left|3-\frac{1}{2}x-1\right|<\varepsilon \qquad |x-4|<\delta$

$\left|2-\frac{1}{2}x\right|<\varepsilon$

$\left|\frac{4-x}{2}\right|<\varepsilon$

$\left|\frac{-x+4}{2}\right|<\varepsilon$

$\left|\frac{-(x-4)}{2}\right|<\varepsilon$

$\left|\frac{-1}{2}\right||x-4|<\varepsilon$

$\frac{1}{2}|x-4|<\varepsilon$

$|x-4|<2\varepsilon$

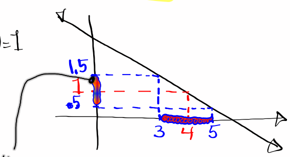
Pick $\delta=2\varepsilon$

Suppose $\varepsilon=\frac{1}{2}$, $\delta=2\left(\frac{1}{2}\right)=1$

Let $x=3.2$

$f(3.2)=3-\frac{1}{2}(3.2)$

$=3-1.6=1.4$



Sep 12-11:04 AM

Use ε and δ definition to prove $\lim_{x \to 1}\frac{2+4x}{3}=2$

$f(x)=\frac{2+4x}{3}$, $L=2$, $a=1$

1) Verify $\lim_{x \to 1}\frac{2+4x}{3}=2$

$\lim_{x \to 1}\frac{2+4x}{3}=\frac{2+4(1)}{3}=\frac{6}{3}=2$ ✓

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$\left|\frac{2+4x}{3}-2\right|<\varepsilon \qquad |x-1|<\delta$

$\left|\frac{2+4x-6}{3}\right|<\varepsilon$

$\left|\frac{4x-4}{3}\right|<\varepsilon$

$\left|\frac{4(x-1)}{3}\right|<\varepsilon$

$\left|\frac{4}{3}\right||x-1|<\varepsilon$

$\frac{4}{3}|x-1|<\varepsilon$

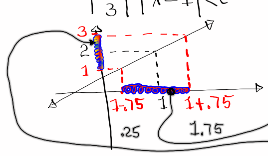
Multiply both sides by $\frac{3}{4}$

$|x-1|<\frac{3\varepsilon}{4}$

$\delta=\frac{3\varepsilon}{4}$

Suppose we choose $\varepsilon=1$, $\delta=\frac{3}{4}=.75$

Pick $x=1.25$



$f(1.25)=\frac{2+4(1.25)}{3}=\frac{7}{3}=2.\overline{3}$

Exam 1: Sep. 21, 2023 (Thursday)

(In-Person) New room in G5.

Sep 12-11:18 AM