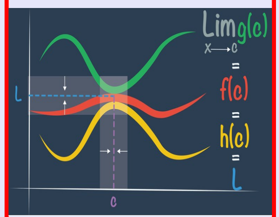
Math 261

Fall 2023

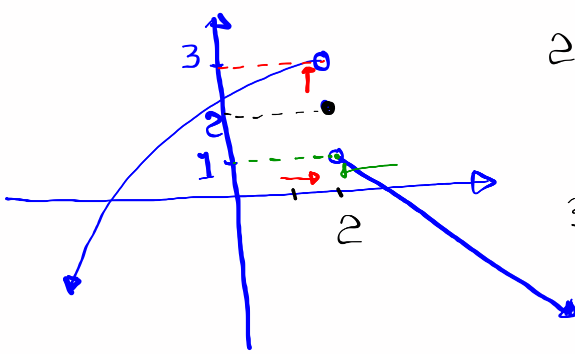
Lecture 8



Feb 19-8:47 AM

Class QZ 6:

Consider the graph of $f(x)$ given below



1) $\lim_{x\to 2^-} f(x) = \boxed{3}$ ✓

Not equal

2) $\lim_{x\to 2^+} f(x) = \boxed{1}$ ✓

3) $\lim_{x\to 2} f(x) = \boxed{D.N.E.}$ ✓

4) $f(2) = \boxed{2}$ ✓

Sep 7-11:22 AM

Evaluate

1) $\lim_{x\to 0} \left[\cos x - \tan x\right]^2 = \left[\cos 0 - \tan 0\right]^2$

$= \left[1 - 0\right]^2 = \boxed{1}$

2) $\lim_{x\to -4} \dfrac{x^3+4x^2}{x^2+4x} = \dfrac{(-4)^3+4(-4)^2}{(-4)^2+4(-4)} = \dfrac{-64+64}{16-16} = \dfrac{0}{0}$ I.F.

$= \lim_{x\to -4} \dfrac{x^2\cancel{(x+4)}}{x\cancel{(x+4)}} = \lim_{x\to -4} x = \boxed{-4}$

$\lim_{x\to -2} \sqrt{x^4+3x+6} = \sqrt{(-2)^4+3(-2)+6}$

$= \sqrt{16 \cancel{-6} \cancel{+6}} = \sqrt{16} = \boxed{4}$

Sep 11-10:28 AM

$\lim_{x\to 0} \dfrac{(x+3)^{-1} - 3^{-1}}{x} = \dfrac{(0+3)^{-1} - 3^{-1}}{0} = \dfrac{3^{-1}-3^{-1}}{0} = \dfrac{0}{0}$ I.F.

$= \lim_{x\to 0} \dfrac{\frac{1}{x+3} - \frac{1}{3}}{x} = \lim_{x\to 0} \dfrac{\cancel{(x+3)}\cdot 3 \cdot \frac{1}{\cancel{x+3}} - (x+3)\cdot \cancel{3} \cdot \frac{1}{\cancel{3}}}{x(x+3)\cdot 3}$

LCD $= (x+3)\cdot 3$

$= \lim_{x\to 0} \dfrac{3-(x+3)}{3x(x+3)}$

$= \lim_{x\to 0} \dfrac{\cancel{3} - \cancel{x}^{\,1} \cancel{-3}}{3\cancel{x}(x+3)}$

$= \lim_{x\to 0} \dfrac{-1}{3(x+3)} = \dfrac{-1}{3(0+3)}$

$= \boxed{\dfrac{-1}{9}}$

Sep 11-10:35 AM

$\lim_{x \to 4} f(x)$ is $4x-9 \le f(x) \le x^2-4x+7$

for $x \ge 0$.

$\lim_{x \to 4}(4x-9) = 4(4)-9 = \boxed{7}$

$\lim_{x \to 4}(x^2-4x+7) = 4^2-4(4)+7 = \boxed{7}$

By Squeeze theorem, $\lim_{x \to 4} f(x) = \boxed{7}$

Sep 11-10:40 AM

Prove $\lim_{x \to 0} x^2 \cos\frac{\alpha}{x} = 0$

Hint:

$-1 \le \cos\alpha \le 1$

So $-1 \le \cos\frac{\alpha}{x} \le 1$

Multiply by $x^2 \ge 0$

$-x^2 \le x^2\cos\frac{\alpha}{x} \le x^2$

$\lim_{x \to 0}(-x^2) = \lim_{x \to 0} x^2 = 0$ by S.T. $\lim_{x \to 0} x^2\cos\frac{\alpha}{x} = \boxed{0}$

Sep 11-10:43 AM

Given $f(x) = \sqrt{x}$

Evaluate $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a} = \lim_{x \to a} \dfrac{\sqrt{x} - \sqrt{a}}{x - a}$

By direct Subs.

$= \dfrac{\sqrt{a} - \sqrt{a}}{a - a} = \dfrac{0}{0}$

I.F.

$$= \lim_{x \to a} \dfrac{\overbrace{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}^{x - a}}{(x - a)(\sqrt{x} + \sqrt{a})}$$

$$= \lim_{x \to a} \dfrac{\cancel{x - a}}{\cancel{(x - a)}(\sqrt{x} + \sqrt{a})} = \lim_{x \to a} \dfrac{1}{\sqrt{x} + \sqrt{a}} = \dfrac{1}{\sqrt{a} + \sqrt{a}}$$

$$= \boxed{\dfrac{1}{2\sqrt{a}}}$$

Sep 11-10:47 AM

Evaluate $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ for $f(x) = x^4$

$$= \lim_{h \to 0} \dfrac{(x+h)^4 - x^4}{h} = \dfrac{(x+0)^4 - x^4}{0} = \dfrac{0}{0}$$

I.F.

$$= \lim_{h \to 0} \dfrac{\left[(x+h)^2\right]^2 - \left[x^2\right]^2}{h}$$

$$= \lim_{h \to 0} \dfrac{\left[(x+h)^2 + x^2\right]\left[(x+h)^2 - x^2\right]}{h} = \lim_{h \to 0} \dfrac{\left[(x+h)^2 + x^2\right]\left[(x+h+x)\right]\left[\cancel{x+h-x}\right]^{1}}{\cancel{h}}$$

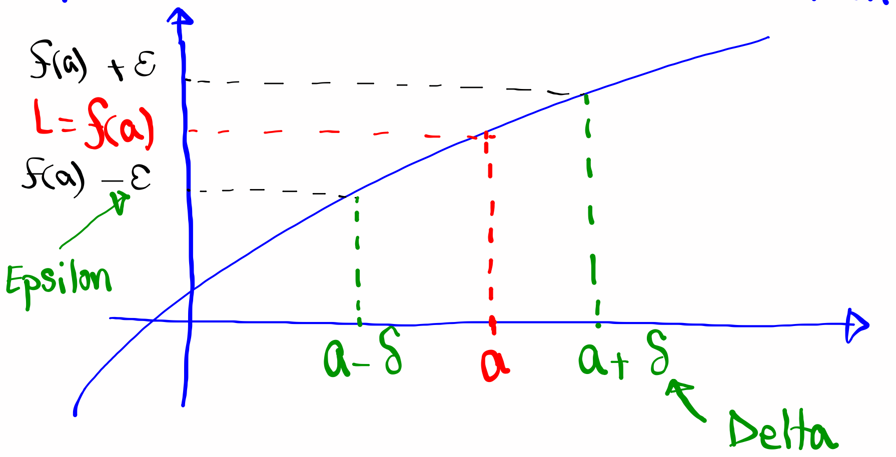
$$= \lim_{h \to 0} \left[(x+h)^2 + x^2\right]\left[x + h + x\right]$$

$$= \left[(x+0)^2 + x^2\right]\left[x + 0 + x\right]$$

$$= 2x^2 \cdot 2x = \boxed{4x^3}$$

Sep 11-10:52 AM

The Precise definition of a limit:



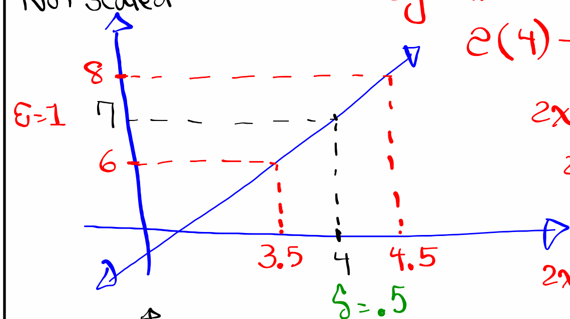
For every $\varepsilon > 0$, there is a $\delta > 0$ such that if $|x-a|<\delta$ then $|f(x)-L|<\varepsilon$.

Sep 11-10:58 AM

Prove $\lim_{x\to 4}(2x-1)=7$

Not Scaled

By direct subs. $2(4)-1=7$ ✓



$2x-1=6$

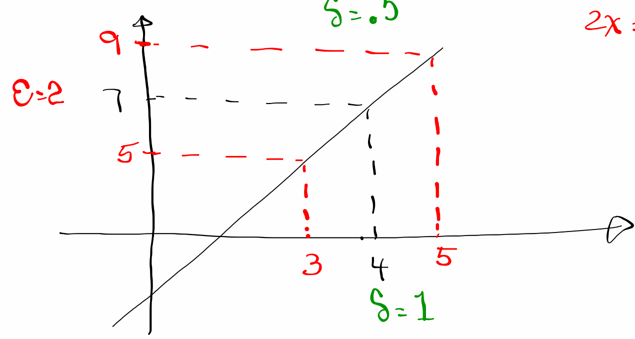
$2x=7 \quad x=\frac{7}{2}$

$=3.5$

$2x-1=8$

$2x=9 \quad x=\frac{9}{2}$

$x=4.5$



Sep 11-11:03 AM

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|2x - 1 - 7| < \varepsilon$ 〃 $|x - 4| < \delta$

$|2x - 8| < \varepsilon$ 〃 $|x - 4| < \delta$

$|2(x - 4)| < \varepsilon$ 〃 $|x - 4| < \delta$

$2|x - 4| < \varepsilon$ 〃 $|x - 4| < \delta$

$|x - 4| < \frac{\varepsilon}{2}$ 〃 $|x - 4| < \delta$

$\delta = \frac{\varepsilon}{2}$

$\varepsilon = 1 \rightarrow \delta = \frac{1}{2} = .5$

$\varepsilon = 2 \rightarrow \delta = \frac{2}{2} = 1$

Sep 11-11:09 AM

Prove $\lim_{x \to 2} \left(\frac{1}{2}x + 3\right) = 4$ ✓

By direct Subs.,

$\frac{1}{2}(2) + 3 = 1 + 3 = 4$ ✓

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{2}x + 3 - 4\right| < \varepsilon$ 〃 $|x - 2| < \delta$

$\left|\frac{1}{2}x - 1\right| < \varepsilon$ 〃 $|x - 2| < \delta$

$\left|\frac{x - 2}{2}\right| < \varepsilon$ 〃 $|x - 2| < \delta$

$|x - 2| < 2\varepsilon$ 〃 $|x - 2| < \delta$

$\delta = 2\varepsilon$

If $\varepsilon = 1$, $\delta = 2$

If $\varepsilon = \frac{1}{2}$, $\delta = 2\left(\frac{1}{2}\right) = 1$

Sep 11-11:14 AM