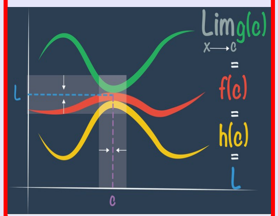
Math 261

Fall 2023

Lecture 7



Feb 19-8:47 AM

Class QZ 5

Evaluate $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = \frac{2^3 - 8}{2^2 - 4} = \frac{8 - 8}{4 - 4} = \boxed{\frac{0}{0}}$ I.F.

Box Your final Ans.

$$\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}(x + 2)} = \lim_{x \to 2} \frac{x^2 + 2x + 4}{x + 2}$$

$$= \frac{2^2 + 2(2) + 4}{2 + 2} = \frac{4 + 4 + 4}{4} = \frac{12}{4} = \boxed{3}$$

Sep 6-11:18 AM

Given $f(x) = x^3$

Evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \frac{(x+0)^3 - x^3}{0} = \frac{0}{0} \text{ I.F.}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim_{h \to 0} (3x^2 + 3xh + h^2)$$

$$= 3x^2 + 3x(0) + 0^2 = \boxed{3x^2}$$

Sep 7-10:29 AM

Evaluate $\lim_{x \to 0} \frac{\sqrt{x^2+36} - 6}{x^2} = \frac{\sqrt{0^2+36} - 6}{0} = \frac{6-6}{0} = \frac{0}{0}$

I.F.

$$\lim_{x \to 0} \frac{(\sqrt{x^2+36} - 6)(\sqrt{x^2+36} + 6)}{x^2(\sqrt{x^2+36} + 6)}$$

$$= \lim_{x \to 0} \frac{x^2 \cancel{+36} \cancel{-36}}{x^2(\sqrt{x^2+36} + 6)} = \lim_{x \to 0} \frac{1}{\sqrt{x^2+36} + 6} = \frac{1}{\sqrt{0^2+36} + 6}$$

$$= \boxed{\frac{1}{12}}$$

Sep 7-10:33 AM

Limit Laws: Assume $\lim_{x\to a} f(x)$ & $\lim_{x\to a} g(x)$ exist

1) $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

2) $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

3) $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$

Given $\begin{cases} \lim_{x\to a} [2f(x) + g(x)] = 5 \\ \lim_{x\to a} [f(x) - 3g(x)] = -1 \end{cases}$ System of equations

Find $\lim_{x\to a} f(x)$ & $\lim_{x\to a} g(x)$.

$3\begin{cases} 2\lim_{x\to a} f(x) + \lim_{x\to a} g(x) = 5 \\ \lim_{x\to a} f(x) - 3\lim_{x\to a} g(x) = -1 \end{cases} \to \begin{cases} 6\lim_{x\to a} f(x) + \cancel{3\lim_{x\to a} g(x)} = 15 \\ \lim_{x\to a} f(x) - \cancel{3\lim_{x\to a} g(x)} = -1 \end{cases}$

$7\lim_{x\to a} f(x) = 14$

$\boxed{\lim_{x\to a} f(x) = 2}$

Plug it back in

$2\lim_{x\to a} f(x) + \lim_{x\to a} g(x) = 5$

$2(2) + \lim_{x\to a} g(x) = 5$

$\boxed{\lim_{x\to a} g(x) = 1}$

Sep 7-10:39 AM

More limit laws:

4) $\lim_{x\to a} [f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

5) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$; $\lim_{x\to a} g(x) \neq 0$

Suppose $\lim_{x\to a} f(x) = 5$, $\lim_{x\to a} g(x) = -2$

Find $\lim_{x\to a} \frac{f(x) \cdot g(x)}{f(x) - g(x)} = \frac{\lim_{x\to a} [f(x) \cdot g(x)]}{\lim_{x\to a} [f(x) - g(x)]}$

$= \frac{\lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)}{\lim_{x\to a} f(x) - \lim_{x\to a} g(x)} = \frac{5(-2)}{5-(-2)} = \boxed{\frac{-10}{7}}$

Sep 7-10:48 AM

More limit laws:

6) $\lim\limits_{x\to a} [f(x)]^n = [\lim\limits_{x\to a} f(x)]^n$, n is a pos. integer

7) $\lim\limits_{x\to a} C = C$

8) $\lim\limits_{x\to a} x = a$

Suppose $\lim\limits_{x\to 2} f(x) = 10$

Find $\lim\limits_{x\to 2} [(f(x))^2 + x - 98] = \lim\limits_{x\to 2} [f(x)]^2 + \lim\limits_{x\to 2} x - \lim\limits_{x\to 2} 98$

$= [\lim\limits_{x\to 2} f(x)]^2 + \lim\limits_{x\to 2} x - \lim\limits_{x\to 2} 98$

$= 10^2 + 2 - 98 = \boxed{4}$

Sep 7-10:54 AM

9) $\lim\limits_{x\to a} x^n = a^n$ n is a pos. integer.

10) $\lim\limits_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ n is a pos. integer
If n is even, then $a \geq 0$

Evaluate $\lim\limits_{x\to -1} [x^3 - \sqrt[3]{x}] = \lim\limits_{x\to -1} x^3 - \lim\limits_{x\to -1} \sqrt[3]{x}$

$= (-1)^3 - \sqrt[3]{-1}$

$= -1 + 1 = \boxed{0}$

Sep 7-10:59 AM

Direct Subs.:

If a is in the domain of $f(x)$, then

$$\lim_{x \to a} f(x) = f(a)$$

Evaluate $\lim_{x \to 0} \frac{x+2}{x^3+8}$

Domain
$x^3+8 \neq 0 \to x \neq -2$
So 0 is in the domain

$$= \frac{0+2}{0^3+8} = \frac{2}{8} = \boxed{\frac{1}{4}}$$

Sep 7-11:03 AM

Evaluate

$$\lim_{x \to 4} \frac{\frac{1}{x} - \frac{1}{4}}{x-4} = \frac{\frac{1}{4} - \frac{1}{4}}{4-4} = \frac{0}{0} \text{ I.F.}$$

LCD $= 4x$

$$= \lim_{x \to 4} \frac{4x \cdot \frac{1}{x} - 4x \cdot \frac{1}{4}}{4x(x-4)} = \lim_{x \to 4} \frac{\overset{-1}{\cancel{4-x}}}{4x\cancel{(x-4)}}$$

$$= \lim_{x \to 4} \frac{-1}{4x} = \boxed{\frac{-1}{16}}$$

Sep 7-11:06 AM

Find $\lim\limits_{x \to 0} \left[\frac{1}{x} - \frac{1}{x^2+x} \right]$

0 is not in the domain

$= \lim\limits_{x \to 0} \left[\frac{1(x+1)}{x(x+1)} - \frac{1}{x(x+1)} \right]$

$= \lim\limits_{x \to 0} \frac{x \cancel{+1} \cancel{-1}}{x(x+1)} = \lim\limits_{x \to 0} \frac{1}{x+1} = \frac{1}{0+1}$

$= \boxed{1}$

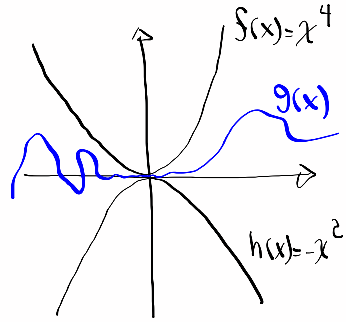
Sep 7-11:10 AM

The Squeeze Theorem

If $f(x) \le g(x) \le h(x)$ when x is near a

and $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} h(x) = L$

then $\lim\limits_{x \to a} g(x) = L$



$-x^2 \le g(x) \le x^4$

$\lim\limits_{x \to 0} (-x^2) = 0$ $\qquad$ $\lim\limits_{x \to 0} x^4 = 0$

$\lim\limits_{x \to 0} g(x) = 0$

Sep 7-11:15 AM

Suppose $2x \le f(x) \le x^4 - x^2 + 2$ for all x, $\forall x$

Find

$$\lim_{x \to 1} f(x)$$

$$\lim_{x \to 1} 2x = 2$$

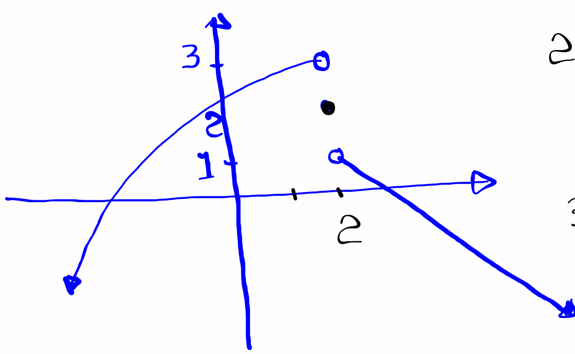
$$\lim_{x \to 1} (x^4 - x^2 + 2) = 2$$

by S.T. $\boxed{\lim_{x \to 1} f(x) = 2}$

Sep 7-11:19 AM

Class QZ 6:

Consider the graph of $f(x)$ given below



1) $\lim_{x \to 2^-} f(x) = \boxed{3}$

2) $\lim_{x \to 2^+} f(x) = \boxed{1}$

3) $\lim_{x \to 2} f(x) = \boxed{\text{D.N.E.}}$

4) $f(2) = \boxed{2}$

Sep 7-11:22 AM