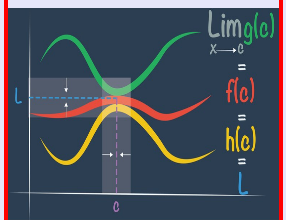
Math 261

Fall 2023

Lecture 6



Class QZ 4

Consider $\boxed{f(x) = \frac{1}{x}}$

Simplify the difference quotient, the evaluate for $h=0$. **Box Your final Answer.**

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{x-(x+h)}{h\,x(x+h)} = \frac{\cancel{x}-\cancel{x}-h}{hx(x+h)}$$

LCD $= x(x+h)$

$$= \frac{-\cancel{h}}{\cancel{h}x(x+h)} = \boxed{\frac{-1}{x(x+h)}}$$

For $h=0$ $\quad \frac{-1}{x(x+0)} = \frac{-1}{x\cdot x} = \frac{-1}{x^2}$

Introduction to limits:

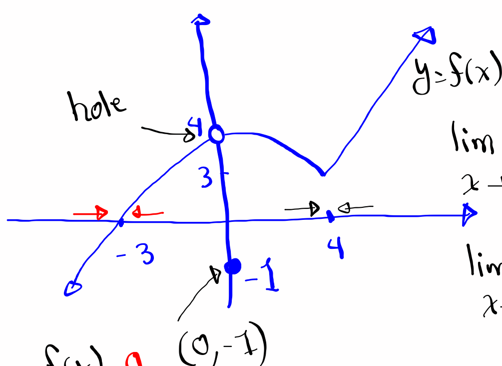
Given $f(x)$

as $x \to a^+ \Rightarrow \lim_{x \to a^+} f(x) = L_1$

as $x \to a^- \Rightarrow \lim_{x \to a^-} f(x) = L_2$

If $L_1 = L_2 \Rightarrow \boxed{\lim_{x \to a} f(x) = L}$

Limit does not mean that $f(a)$ is defined.



$\lim_{x \to 4^+} f(x) = 3$

$\lim_{x \to 4^-} f(x) = 3$

So $\lim_{x \to 4} f(x) = 3$

$\lim_{x \to -3^-} f(x) = 0$

$\lim_{x \to -3^+} f(x) = 0$

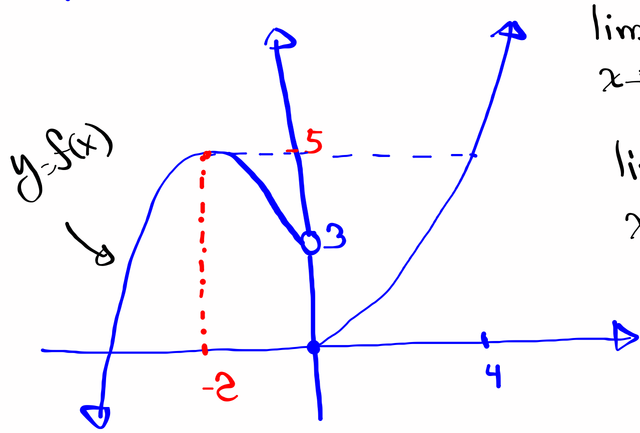
$\Rightarrow \lim_{x \to -3} f(x) = 0$

$\lim_{x \to 0^+} f(x) = 4$ $\quad \lim_{x \to 0^-} f(x) = 4$

$\lim_{x \to 0} f(x) = 4$

$f(0) = -1$

Consider the function below



$\lim\limits_{x\to -2^-} f(x) = 5$

$\lim\limits_{x\to -2^+} f(x) = 5$

$\lim\limits_{x\to -2} f(x) = 5$

$\lim\limits_{x\to 0^+} f(x) = \boxed{0}$

$\lim\limits_{x\to 0} f(x) =$ D.N.E.

$f(-2) = 5$

why not? $\lim\limits_{x\to a^+} f(x) \neq \lim\limits_{x\to a^-} f(x)$

$\lim\limits_{x\to 0^-} f(x) = \boxed{3}$

$f(0) = \boxed{0}$

How to evaluate the limit?

Always plug in the value, hope for the best.

$$\lim_{x\to -5} (x^2 - 4x) = (-5)^2 - 4(-5) = 25 + 20 = \boxed{45}$$

$$\lim_{x\to\sqrt{2}} \frac{2x}{x^2-1} = \frac{2(\sqrt{2})}{(\sqrt{2})^2 - 1} = \frac{2\sqrt{2}}{2-1} = \frac{2\sqrt{2}}{1} = \boxed{2\sqrt{2}}$$

45°

$$\lim_{x\to\frac{\pi}{4}} (\tan x + \cot x) = \tan\frac{\pi}{4} + \cot\frac{\pi}{4}$$

$$= 1 + 1 = \boxed{2}$$

what happens if we plug in the value but we have a situation?

$$\lim_{x\to 1} \frac{2x^2-2x}{x-1} = \frac{2(1)^2-2(1)}{1-1} = \frac{2-2}{1-1} = \boxed{\frac{0}{0}}$$

Indeterminate form

we can try factoring to simplify

$$\lim_{x\to 1} \frac{2x^2-2x}{x-1} = \lim_{x\to 1} \frac{2x\cancel{(x-1)}}{\cancel{x-1}} = \lim_{x\to 1} 2x = 2(1) = \boxed{2}$$

Evaluate $\lim_{x\to 3} \frac{x^2-5x+6}{x^2-9} = \frac{3^2-5(3)+6}{3^2-9}$

$$= \frac{9-15+6}{9-9} = \frac{0}{0} \text{ I.F.}$$

Try Factoring

$$\lim_{x\to 3} \frac{x^2-5x+6}{x^2-9} = \lim_{x\to 3} \frac{(x-2)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} = \lim_{x\to 3} \frac{x-2}{x+3} = \frac{3-2}{3+3}$$

$$= \boxed{\frac{1}{6}}$$

Evaluate $\lim_{x\to 3} \dfrac{\frac{1}{x-2} - 1}{x-3} = \dfrac{\frac{1}{3-2} - 1}{3-3} = \dfrac{\frac{1}{1} - 1}{3-3}$

$= \dfrac{1-1}{3-3} = \dfrac{0}{0}$ I.F.

Try using LCD to clear/Simplify

LCD $= x-2$

$$\lim_{x\to 3} \frac{\frac{1}{x-2} - 1}{x-3} = \lim_{x\to 3} \frac{\cancel{(x-2)}\cdot\frac{1}{\cancel{x-2}} - (x-2)\cdot 1}{(x-2)(x-3)} = \lim_{x\to 3} \frac{1-(x-2)}{(x-2)(x-3)}$$

$$= \lim_{x\to 3} \frac{1-x+2}{(x-2)(x-3)} = \lim_{x\to 3} \frac{-x+3}{(x-2)(x-3)} = \lim_{x\to 3} \frac{-\cancel{(x-3)}}{(x-2)\cancel{(x-3)}}$$

$$= \lim_{x\to 3} \frac{-1}{x-2} = \frac{-1}{3-2} = \frac{-1}{1} = \boxed{-1}$$

Evaluate $\lim_{x\to 4} \dfrac{\sqrt{x} - 2}{x-4} = \dfrac{\sqrt{4} - 2}{4-4} = \dfrac{2-2}{4-4} = \dfrac{0}{0}$

I.F.

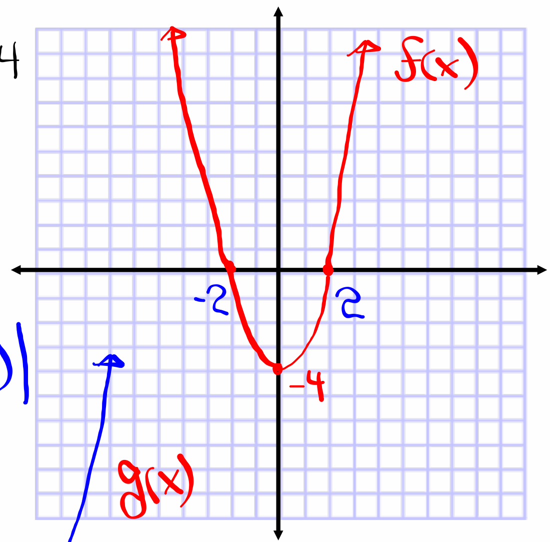
One way to proceed is to rationalize the num.

$$\lim_{x\to 4} \frac{\sqrt{x} - 2}{x-4} = \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}$$

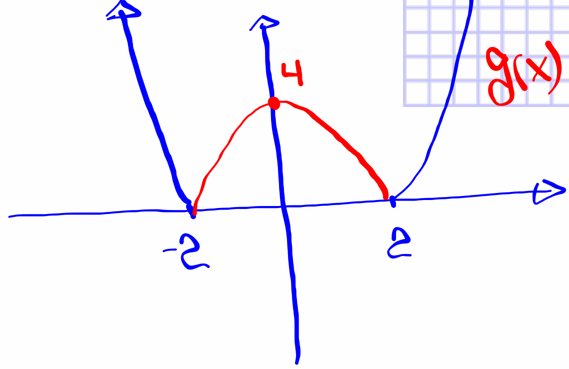
$$= \lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2}$$

$$= \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

Graph $f(x) = x^2 - 4$



Graph $g(x) = |f(x)|$



Special Factoring

$A^2 - B^2 = (A - B)(A + B)$

$A^2 + B^2 = \text{Prime}$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

Class QZ 5

Evaluate $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = \frac{2^3 - 8}{2^2 - 4} = \frac{8 - 8}{4 - 4} = \frac{0}{0}$ I.F.

Box Your final Ans.

$$\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}(x + 2)} = \lim_{x \to 2} \frac{x^2 + 2x + 4}{x + 2}$$

$$= \frac{2^2 + 2(2) + 4}{2 + 2} = \frac{4 + 4 + 4}{4} = \frac{12}{4} = \boxed{3}$$