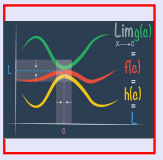
Math 261

Fall 2023

Lecture 58



Feb 19-8:47 AM

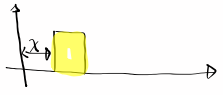
Introduction To Work:

Work required to move an object from a to b is given by

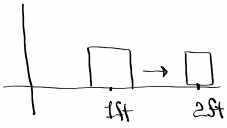
$$W = \int_a^b f(x)\,dx$$

$f(x)$ is the force applied to move the object.

An object is located x feet from the origin. A force of x^3+x pounds acts on it.



How much work is required to move it from 1ft to 2ft.

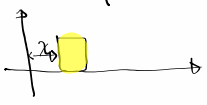


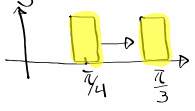
$$W = \int_1^2 \underbrace{(x^3+x)}_{\text{force}}\,dx$$

$$= \left(\frac{x^4}{4} + \frac{x^2}{2}\right)\Big|_1^2 = \boxed{5.25 \text{ ft/lb}}$$

Dec 13-10:27 AM

An object is x feet from the origin.
A force of $\sec^2 x$ pounds acts on it.
How much is required to move it from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$.





$$W = \int_{\pi/4}^{\pi/3} \sec^2 x \, dx$$

$$= \tan x \Big|_{\pi/4}^{\pi/3}$$

$$= \tan\frac{\pi}{3} - \tan\frac{\pi}{4}$$

$$= (\sqrt{3} - 1) \text{ ft/lb.}$$

Dec 13-10:33 AM

Hook's Law:

It is the force applied to a spring to stretch it

$$f(x) = Kx$$

K is the spring constant, $K > 0$

x cannot be too large.

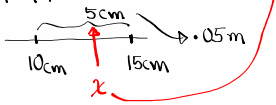
Dec 13-10:37 AM

A Spring has a length of 10 cm.

A force of 40 N is applied to it to

Newton

Stretch it to 15 cm.



$f(x) = Kx$

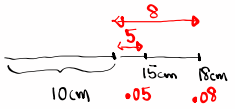
$40 = K \cdot (.05)$

$\frac{40}{.05} = K$

$K = 800$

$f(x) = 800x$

How much work is required to stretch from 15 cm to 18cm



$W = \int_{.05}^{.08} 800x\,dx$

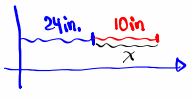
$= 800\,\frac{x^2}{2}\Big|_{.05}^{.08} = 400(.08^2 - .05^2)$

$= 1.56\text{ J}$

Dec 13-10:40 AM

A Spring has a length of 24 inches.

A force of 5 lb. applies to it to stretch it 10 inches beyond its natural length of 24in.

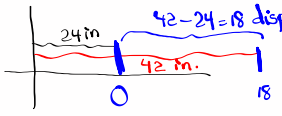


$f(x) = Kx$ ← Displacement

$5 = K \cdot 10$ $\quad K = \frac{1}{2}$

$f(x) = \frac{1}{2}x$

How much is required it to stretch it from Natural length To 42 inches long.



42 − 24 = 18 displacement

$W = \int_0^{18} \frac{1}{2}x\,dx$

$= \frac{1}{2} \cdot \frac{x^2}{2}\Big|_0^{18}$

$= \frac{1}{4}(18^2 - 0^2) = 81\text{ in./lb.}$

You have to empty it water is being displaced

You need a device to do the work.

Dec 13-10:48 AM

Final Exam:

Arrive early, if room is available, we start.

I will give you additional reasonable time if needed.

Review lectures from last 3-weeks.

Review exams 1, 2, and 3.

make 2-Page cheat sheet, front & back.

Finish Strong. Do not leave any Problem blank

Dec 13-10:59 AM

$$f(x) = \int_0^{x^2} \frac{1}{1+\sin\sqrt{t}}\,dt$$

$$f(0) = \int_0^{0^2} \frac{1}{1+\sin\sqrt{t}}\,dt = \int_0^0 \frac{1}{1+\sin\sqrt{t}}\,dt = 0$$

$$f'(x) = \frac{1}{1+\sin\sqrt{x^2}} \cdot 2x - \frac{1}{1+\sin\sqrt{0}} \cdot 0$$

$$f'(x) = \frac{2x}{1+\sin x} \qquad f'(0) = \frac{2(0)}{1+\sin 0} = 0$$

$$f''(x) = \frac{2(1+\sin x) - 2x\cdot\cos x}{(1+\sin x)^2} \qquad f''(0) = \boxed{2}$$

Dec 13-11:08 AM