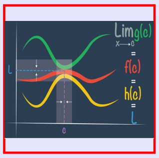
Math 261

Fall 2023

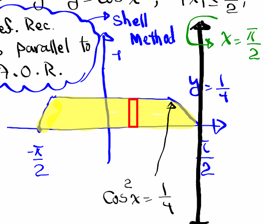
Lecture 57



Feb 19-8:47 AM

Draw the region bounded by $y = \cos^2 x$, $|x| \le \frac{\pi}{2}$, and $y = \frac{1}{4}$.

Ref. Rec. is parallel to A.O.R. → Shell Method



$|x| \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\cos \frac{\pi}{2} = 0$, $\cos -\frac{\pi}{2} = 0$

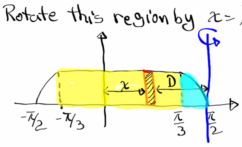
$\cos 0 = 1$

$\cos x = \frac{1}{2}$, $\cos x = \frac{-1}{2}$

Ref. Angle

$\pi/3$

Rotate this region by $x = \pi/2$

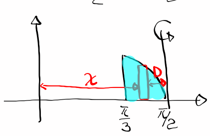


Shell

$2\pi D H$

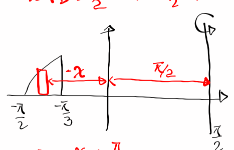
$x + D = \frac{\pi}{2}$ $\quad D = \frac{\pi}{2} - x$

$$\int_{-\pi/3}^{\pi/3} 2\pi\left(\frac{\pi}{2} - x\right)\frac{1}{4}\, dx$$



$$\int_{\pi/3}^{\pi/2} 2\pi\left(\frac{\pi}{2} - x\right)\cos^2 x\, dx$$

$x + D = \frac{\pi}{2}$ $\quad D = \frac{\pi}{2} - x$



$$\int_{-\frac{\pi}{2}}^{-\frac{\pi}{3}} 2\pi\left(-x + \frac{\pi}{2}\right)\cos^2 x\, dx$$

$D = -x + \frac{\pi}{2}$

Dec 12-10:25 AM

Find f_{ave} for $f(x) = (x-3)^2$ on $[2,4]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$u = x-3$ $\quad$ $x=2$, $u=-1$
$du = dx$ $\quad$ $x=4$, $u=1$

$$f_{ave} = \frac{1}{4-2}\int_2^4 (x-3)^2\,dx = \frac{1}{2}\int_{-1}^{1} u^2\,dx$$

when $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if $f(x)$ is even.

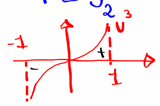
$$= \frac{1}{\cancel{2}}\cdot\cancel{2}\int_0^1 u^2\,du$$

$$= \frac{u^3}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$$

Find f_{ave} for $f(x) = (x-3)^3$ on $[2,4]$

$$f_{ave} = \frac{1}{4-2}\int_2^4 (x-3)^3\,dx = \frac{1}{2}\int_{-1}^{1} u^3\,du = \boxed{0}$$

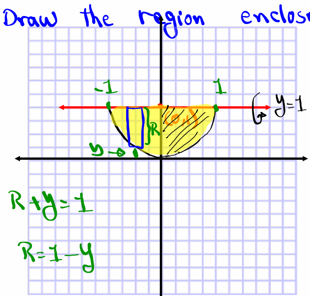
odd



Dec 12-10:44 AM

Draw the region enclosed by $x^2 + (y-1)^2 = 1$.

Circle $(y-1)^2 = 1 - x^2$
$(0,1)$, $r = 1$



Rotate this region about $y=1$

Ref. Rect. $\perp$ A.O.R.

Region attached 100% to A.O.R.

DISK Method

$\pi R^2\,\Delta x$

$R + y = 1$

$R = 1 - y$

$$V = \int_{-1}^{1} \pi(1-y)^2\,dx = 2\int_0^1 \pi\,(y-1)^2\,dx$$

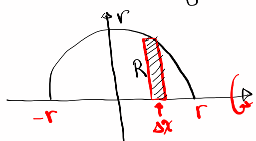
$(1-y)^2 = (y-1)^2$

$$= 2\pi\int_0^1 (1-x^2)\,dx$$

$$= 2\pi\left[x - \frac{x^3}{3}\right]\Big|_0^1 = 2\pi\cdot\frac{2}{3} = \frac{4\pi}{3}$$

Dec 12-10:54 AM

Consider the region bounded by $x^2+y^2=r^2$. Rotate about x-axis.



$R=y=\sqrt{r^2-x^2}$

$R^2=y^2=r^2-x^2$

$$\int_{-r}^{r}\pi(R)^2\,dx = 2\int_0^r \pi(r^2-x^2)\,dx$$

Disk method

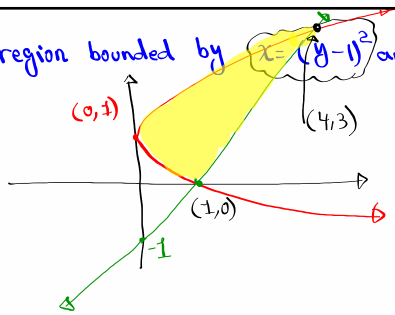
$$=2\pi\left[r^2x-\frac{x^3}{3}\right]\Big|_0^r$$

$$=2\pi\left[r^3-\frac{r^3}{3}\right]=2\pi\cdot\frac{2r^3}{3}$$

$$=\boxed{\frac{4\pi r^3}{3}}$$ volume of Sphere

Dec 12-11:03 AM

Draw the region bounded by $x=(y-1)^2$ and $x-y=1$



$$\begin{cases} x-y=1 \\ x=(y-1)^2 \end{cases}$$

$(y-1)^2-y=1$

$y^2-2y+1-y=1$

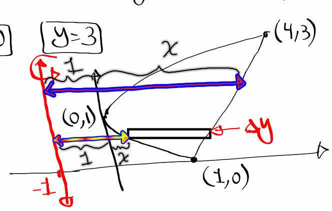
$y^2-3y=0$ $\boxed{y=0}$ $\boxed{y=3}$

$y=0 \to x=1$

$y=3 \to x=4$

Rotate about $x=-1$

washer



$$V=\int_0^3 \pi[R^2-r^2]\,dy$$

$R=1+x=1+y+1=y+2$

$r=1+(y-1)^2$

$$V=\pi\int_0^3\left[(y+2)^2-(1+(y-1)^2)^2\right]dy$$

Ans: $\frac{117\pi}{5}$

Dec 12-11:10 AM

Evaluate $\int x^2\sqrt{2+x}\,dx$

$u=\sqrt{2+x}$

$u^2=2+x \rightarrow u^2-2=x$

$2u\,du=dx$

$=\int (u^2-2)^2 u \; 2u\,du$

$=2\int (u^4-4u^2+4)\cdot u^2\,du$

$=2\int (u^6-4u^4+4u^2)\,du$

$=2\left[\frac{u^7}{7}-\frac{4u^5}{5}+\frac{4u^3}{3}\right]+C$

$=2\left[\frac{(\sqrt{2+x})^7}{7}-\frac{4(\sqrt{2+x})^5}{5}+\frac{4(\sqrt{2+x})^3}{3}\right]+C$

Dec 12-11:22 AM

Evaluate

$\int x^3\sqrt{x^2+1}\,dx$

$u=\sqrt{x^2+1}$

$u^2=x^2+1$

$2u\,du=2x\,dx$

$u\,du=x\,dx$

$=\int x^2\, x\,\sqrt{x^2+1}\,dx \;=\int (u^2-1)\,u\;u\,du$

$=\int (u^2-1)\,u^2\,du$

$=\frac{u^5}{5}-\frac{u^3}{3}+C$

$\boxed{=\frac{(\sqrt{x^2+1})^5}{5}-\frac{(\sqrt{x^2+1})^3}{3}+C}$

Dec 12-11:28 AM