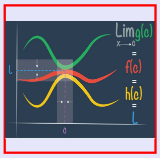
Math 261

Fall 2023

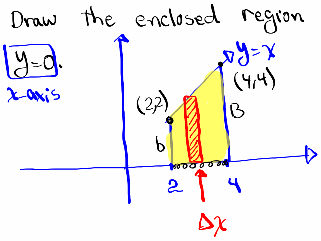
Lecture 54



Feb 19-8:47 AM

Draw the enclosed region by $\boxed{x=2\ ,\ x=4}$, $\boxed{y=x}$, and $\boxed{y=0}$.

x-axis



Trapezoid

$$A = \frac{h(B+b)}{2}$$

$$= \frac{2(4+2)}{2} = \boxed{6}\ \checkmark$$

$$A = \int_2^4 (\text{Top} - \text{Bottom})\,dx$$

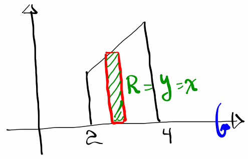
$$= \int_2^4 (x-0)\,dx = \frac{x^2}{2}\Big|_2^4$$

$$= \frac{1}{2}\left[4^2 - 2^2\right] = \frac{1}{2}(16-4)$$

$$= \frac{12}{2} = \boxed{6}\ \checkmark$$

Dec 6-10:25 AM

Rotate that region by x-axis, find the volume.



Ref. Rec. $\perp$ A.O.R.

Region is totally attached to A.O.R.

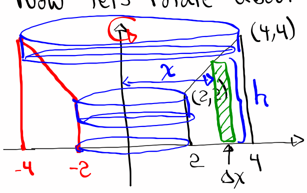
$\Rightarrow$ Disk Method

πR^2

$$\int_2^4 \pi x^2 dx = \pi \frac{x^3}{3}\Big|_2^4 = \frac{\pi}{3}\left(4^3 - 2^3\right) = \frac{\pi}{3}(64-8) = \boxed{\frac{56\pi}{3}}$$

Dec 6-10:30 AM

Now let's rotate about y-axis, find the volume.



Ref. Rect. is parallel to A.O.R.

Shell Method

$2\pi \cdot$ Distance from A.O.R. $\cdot$ Height of Ref. Rec

$$V = \int_2^4 2\pi \cdot x \cdot (x-0)\,dx$$

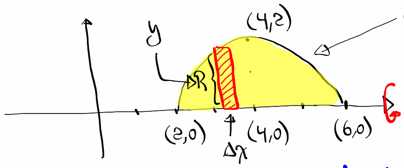
$$= 2\pi \int_2^4 x^2 dx = 2\pi \cdot \frac{x^3}{3}\Big|_2^4 = \frac{2\pi}{3}\left(4^3 - 2^3\right)$$

$$= \frac{2\pi}{3}(64-8) = \frac{2\pi}{3} \cdot 56$$

$$= \boxed{\frac{112\pi}{3}}$$

Dec 6-10:34 AM

Draw the top-half of a Circle with radius 2 and Centered at (4,0).



$(x-h)^2+(y-k)^2=R^2$

$(x-4)^2+(y-0)^2=2^2$

$(x-4)^2+y^2=4$

$y^2=4-(x-4)^2$

$y=\sqrt{4-(x-4)^2}$

Rotate about x-axis and find Volume

Ref. Rect. ⊥ A.O.R. ⇒ Disk Method πR^2

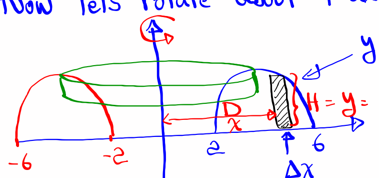
Region is attached 100% to A.O.R.

$$V=\int_2^6 \pi\left(\sqrt{4-(x-4)^2}\right)^2 dx = \pi\int_2^6 \left(4-(x-4)^2\right)dx$$

Sphere $V=\frac{4\pi r^3}{3}=\frac{4\pi(2)^3}{3}=\boxed{\frac{32\pi}{3}}$ $=\boxed{\quad}$

Dec 6-10:40 AM

Now let's rotate about Y-axis.



$y=\sqrt{4-(x-4)^2}$

$H=y=\sqrt{4-(x-4)^2}$

Ref. Rect is Parallel to A.O.R. ⇒ Shell Method $2\pi D H$

Distance from A.O.R.

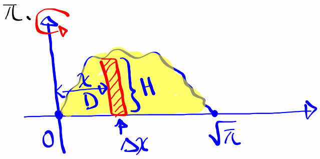
Height of Ref. Rect.

$$V=\int_2^6 2\pi x\cdot\sqrt{4-(x-4)^2}\,dx$$

$\boxed{\text{Try to evaluate this by Thursday}}$

Dec 6-10:49 AM

Consider the enclosed region by $f(x) = \sin x^2$, x-axis, from $x=0$ to $x=\sqrt{\pi}$.



Let's rotate about Y-axis, Find the volume

Ref. Rect. parallel to A.O.R. ⇒ Shell Method

$2\pi \cdot D \cdot H$

$$V = \int_0^{\sqrt{\pi}} 2\pi \cdot x \cdot \sin x^2 \, dx$$

$u = x^2$ $\quad x=0 \to u=0$

$du = 2x\,dx$ $\quad x=\sqrt{\pi} \to u=\pi$

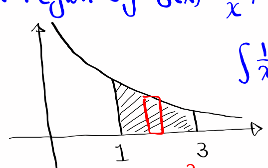
$$= \int_0^{\pi} \pi \sin u \, du = \pi \cdot -\cos u \Big|_0^{\pi} = -\pi\left[\cos\pi - \cos 0\right]$$

$$= \boxed{2\pi}$$

Dec 6-10:54 AM

Draw the enclosed region by $f(x) = \frac{1}{x}$, x-axis, $x=1$ and $x=3$.

$\int \frac{1}{x}dx = \ln x + C$
$x>0$



$$A = \int_1^3 (\text{Top} - \text{Bottom})\,dx = \int_1^3 \left(\frac{1}{x} - 0\right)dx = \int_1^3 \frac{1}{x}\,dx$$

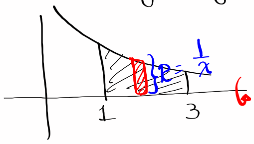
$$= \ln x \Big|_1^3$$

$$= \ln 3 - \cancel{\ln 1}^{\to 0}$$

$$= \boxed{\ln 3} \approx 1.1$$

Dec 6-11:01 AM

Rotate the region by x-axis.



Disk Method

$$V = \int_1^3 \pi \left(\frac{1}{x}\right)^2 dx$$

$$= \pi \int_1^3 x^{-2} dx$$

$$= \pi \cdot \frac{x^{-1}}{-1} \Big|_1^3$$

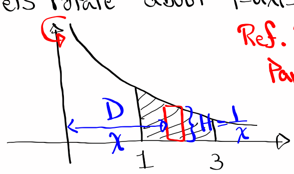
$$= -\pi \cdot \frac{1}{x} \Big|_1^3$$

$$= -\pi \left[\frac{1}{3} - 1\right] = -\pi \cdot \frac{-2}{3}$$

$$= \boxed{\frac{2\pi}{3}}$$

Dec 6-11:06 AM

Now let's rotate about Y-axis.



Ref. Rec. is Parallel to A.O.R.

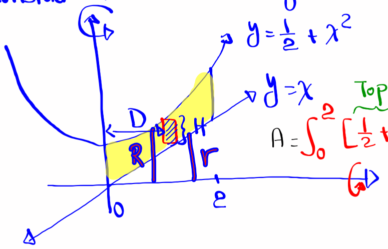
$\Rightarrow$ Shell Method

$2\pi \cdot D \cdot H$

$$V = \int_1^3 2\pi \cdot \cancel{x} \cdot \frac{1}{\cancel{x}} dx = 2\pi \cdot x \Big|_1^3 = 2\pi(3-1) = \boxed{4\pi}$$

Dec 6-11:09 AM

Consider the shaded region below



Top Bottom

$A=\int_0^2\left[\frac{1}{2}+x^2-x\right]dx=\boxed{}$

Rotate about x-axis, Find Volume

Disk
$\boxed{\text{Washer}}$
Shell

$V=\int_0^2\pi\left[R^2-r^2\right]dx$

$=\pi\int_0^2\left[\left(\frac{1}{2}+x^2\right)^2-(x)^2\right]dx=\boxed{}$

Rotate about Y-axis, Find Volume

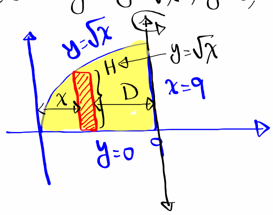
Disk
Washer
$\boxed{\text{Shell}}$

$V=\int_0^2 2\pi(x)\cdot\left(\frac{1}{2}+x^2-x\right)dx=\boxed{}$

Dec 6-11:12 AM

Rotate the region bounded by $y=\sqrt{x}$, $y=0$, and $x=9$

1) about $x=9$



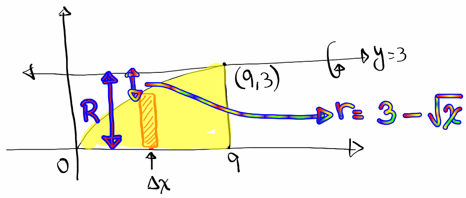
2) about $y=3$.

$x+D=9$
$D=9-x$

Shell
$2\pi\cdot D\cdot H$

$V=\int_0^9 2\pi(9-x)\cdot\sqrt{x}\,dx=\boxed{}$

Dec 6-11:21 AM



Washer

$$V = \int_0^9 \pi\left[R^2 - r^2\right]dx = \pi\int_0^9\left[3^2 - (3-\sqrt{x})^2\right]dx$$

Dec 6-11:27 AM