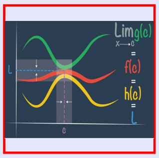
Math 261

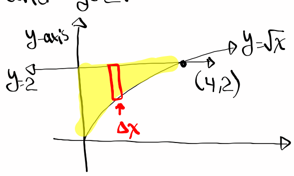
Fall 2023

Lecture 53



Feb 19-8:47 AM

Consider the region bounded by y-axis, $y=\sqrt{x}$, and $y=2$.



$\sqrt{x}=2$

$x=4$

$$\text{Area}=\int_0^4 (\text{Top}-\text{Bottom})\,dx$$

$$=\int_0^4 [2-\sqrt{x}]\,dx \qquad \sqrt{x} \rightarrow x^{1/2}$$

$$=\left(2x-\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right)\Bigg|_0^4$$

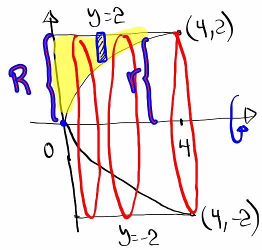
$$=\left(2x-\frac{x^{3/2}}{3/2}\right)\Bigg|_0^4=\left(2x-\frac{2}{3}x\sqrt{x}\right)\Bigg|_0^4$$

$$=2\cdot 4-\frac{2}{3}\cdot 4\sqrt{4}-0$$

$$=8-\frac{16}{3}=\frac{24}{3}-\frac{16}{3}=\boxed{\frac{8}{3}}$$

Dec 5-10:26 AM

Rotate that region by x-axis, find the Volume.



Vertical Ref. Rec.

$$\int_0^4 \pi\left[4^2-(\sqrt{x})^2\right]dx$$

Ref. Rec. $\perp$ A.O.R.

Region is not totally attached to A.O.R.

$\rightarrow$ Washer Method

$$\pi\left[R^2 - r^2\right]$$

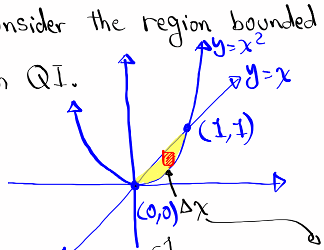
$$V = \pi\int_0^4 (16-x)\,dx$$

$$= \pi\left[16x - \frac{x^2}{2}\right]\Big|_0^4 = \pi\left[\left(16\cdot 4 - \frac{4^2}{2}\right) - (0)\right]$$

$$= \pi\left[64-8\right] = \boxed{56\pi}$$

Dec 5-10:32 AM

Consider the region bounded by $y=x$, and $y=x^2$ in QI.



$x^2 = x$

$x^2 - x = 0$

$x(x-1)=0$

$x=0 \quad x=1$

$$A = \int_0^1 (\text{Top}-\text{Bottom})\,dx$$

$$= \int_0^1 \left[x - x^2\right]dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{3}$$

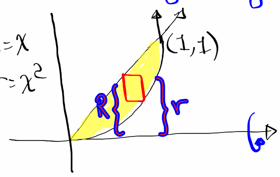
$$= \boxed{\frac{1}{6}}$$

Dec 5-10:39 AM

Rotate that region by x-axis, find the volume.

$R = x$

$r = x^2$



Ref. Rect. $\perp$ x-axis

$$\int_0^1 \pi\left[x^2 - (x^2)^2 \right] dx$$

Ref. Rect. $\perp$ A.O.R.

Region is not totally attached to A.O.R. $\Rightarrow$ washer Method

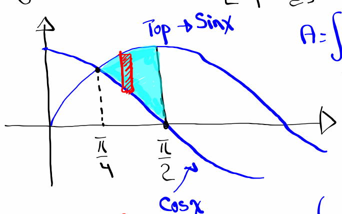
$$\pi\left[R^2 - r^2\right]$$

$$V = \pi\int_0^1 \left[x^2 - x^4\right] dx = \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]\Big|_0^1$$

$$= \pi\left[\frac{1}{3} - \frac{1}{5}\right] = \boxed{\frac{2\pi}{15}}$$

Dec 5-10:44 AM

Consider the region bounded by $y = \sin x$, $y = \cos x$ over $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.



$$A = \int_{\pi/4}^{\pi/2} \left[\text{Top} - \text{Bottom}\right] dx$$

$$= \int_{\pi/4}^{\pi/2} \left[\sin x - \cos x\right] dx$$

$$= \left(-\cos x - \sin x\right)\Big|_{\pi/4}^{\pi/2}$$

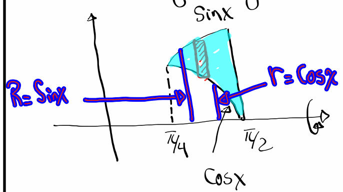
$$= \left(-\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\right) - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$$

$$\cos\frac{\pi}{2} \to 0 \qquad \sin\frac{\pi}{2} \to 1$$

$$= -1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = -1 + \sqrt{2} = \boxed{\sqrt{2} - 1} > 0$$

Dec 5-10:49 AM

Rotate the region by x-axis.



Ref. Rect. ⊥ x-axis

$$\int_{\pi/4}^{\pi/2} \pi\left[\sin^2 x - \cos^2 x\right]dx$$

$$= \int_{\pi/4}^{\pi/2} \pi(-\cos 2x)\,dx$$

⇒ Washer Method

Ref. Rect. ⊥ A.O.R.

Region is not totally attached to A.O.R.

$$\pi\left[R^2 - r^2\right]$$

Recall from Trig

$\cos 2x = \cos^2 x - \sin^2 x$

So

$-\cos 2x = \sin^2 x - \cos^2 x$

$x = \frac{\pi}{4} \rightarrow u = \frac{\pi}{2}$

$x = \pi/2 \rightarrow u = \pi$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$$\int_{\pi/2}^{\pi} \pi\cdot(-\cos u)\cdot\frac{du}{2}$$

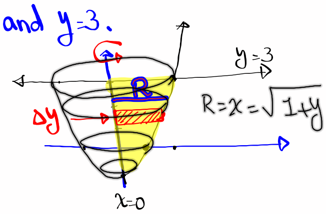
$$= \frac{-\pi}{2}\cdot \sin u \Big|_{\pi/2}^{\pi}$$

$$= \frac{-\pi}{2}\left[\cancelto{0}{\sin \pi} - \cancelto{1}{\sin \frac{\pi}{2}}\right]$$

$$= \frac{-\pi}{2}\cdot(-1) = \boxed{\frac{\pi}{2}}$$

Dec 5-10:56 AM

Draw the region bounded by $x = \sqrt{1+y}$, $\underbrace{x=0}_{\text{Y-axis}}$, and $y=3$.



$y=0 \rightarrow x=1$

$y=3 \rightarrow x=2$

$x^2 = 1+y$

$x^2 - 1 = y$

$\rightarrow x \geq 0$

Rotate about Y-axis, find Volume.

$$V = \int_{-1}^{3} \pi\left[\sqrt{1+y}\right]^2 dy$$

$$= \pi\int_{-1}^{3}(1+y)\,dy$$

$$= \pi\left[y + \frac{y^2}{2}\right]\Big|_{-1}^{3} = \boxed{}$$

Ref. Rect. ⊥ A.O.R.

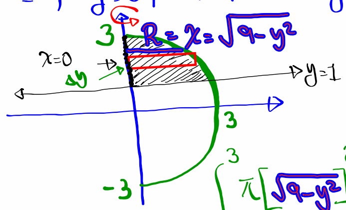
Region is totally attached to A.O.R.

→ disk method

πR^2

Dec 5-11:06 AM

Rotate the region bounded by $x=\sqrt{9-y^2}$, $y=1$, $y=3$, and $x=0$ by y-axis.



$x^2=9-y^2$

$x^2+y^2=9$

Disk Method

$$\int_1^3 \pi\left[\sqrt{9-y^2}\right]^2 dy$$

$$=\pi\int_1^3 (9-y^2)\,dy=\pi\left[9y-\frac{y^3}{3}\right]\Bigg|_1^3$$

$$=\boxed{}$$

Dec 5-11:15 AM

Find x if $\int_0^x \frac{1}{(3t+1)^2}\,dt=\frac{1}{6}$

$$\int_0^x (3t+1)^{-2}\,dt=\int_1^{3x+1} u^{-2}\cdot\frac{du}{3}=\frac{1}{3}\cdot\frac{u^{-1}}{-1}\Bigg|_1^{3x+1}$$

$u=3t+1$

$du=3\,dt$

$\frac{du}{3}=dt$

$$=\frac{-1}{3}\cdot\frac{1}{u}\Bigg|_1^{3x+1}$$

$$=\frac{-1}{3}\left(\frac{1}{3x+1}-1\right)=\frac{1}{6}$$

$$\frac{d}{dx}\left[\int_0^x \frac{1}{(3t+1)^2}\,dt\right]=\frac{d}{dx}\left[\frac{1}{6}\right]$$

$$\frac{1}{(3x+1)^2}\cdot 1-\frac{1}{1}\cdot 0=0$$

$$\frac{1}{(3x+1)^2}=0 \rightarrow \text{NO Solution}$$

Multiply by 6

$$-2\left(\frac{1}{3x+1}-1\right)=1$$

$$\frac{-2}{3x+1}+2=1$$

$$\frac{-2}{3x+1}=-1$$

$$3x+1=2 \quad \boxed{x=\frac{1}{3}}$$

Dec 5-11:23 AM