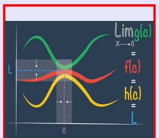
Math 261

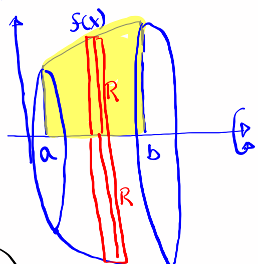
Fall 2023

Lecture 52



Feb 19-8:47 AM

Suppose $f(x) \geq 0$, and Cont. on $[a,b]$. we take the enclosed region below $f(x)$ and above x-axis from a to b and rotate it about x-axis.



Volume

$$V=\int_a^b \pi R^2 \, dx$$

Disk Method

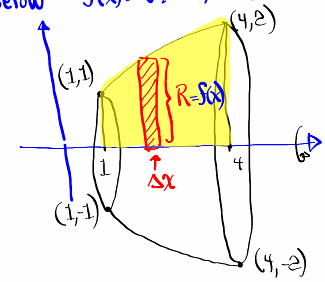
Ref. Rec. $\perp$ A.O.R.

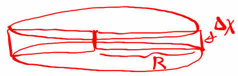
Region is attached 100% to A.O.R.

Dec 4-10:26 AM

Consider the enclosed region for $x=1$ to $x=4$, below $f(x)=\sqrt{x}$, and above the x-axis.

Rotate about x-axis.





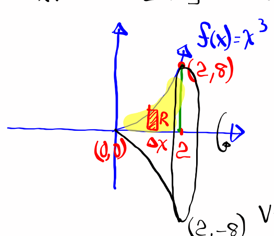
$V=\pi R^2 \Delta x$

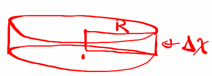
$$V=\int_1^4 \pi(\sqrt{x})^2 dx$$

$$=\pi\int_1^4 x\,dx=\pi\cdot\frac{x^2}{2}\Big|_1^4=\frac{\pi}{2}(4^2-1^2)=\boxed{\frac{15\pi}{2}}$$

Dec 4-10:31 AM

Rotate the enclosed region bounded by $f(x)=x^3$, x-axis on $[0,2]$ about x-axis.





$V=\pi R^2 \Delta x$

$$V=\int_0^2 \pi(x^3)^2 dx$$

$$=\pi\int_0^2 x^6 dx=\pi\cdot\frac{x^7}{7}\Big|_0^2$$

$$=\frac{\pi}{7}(2^7-0^7)$$

$$=\boxed{\frac{128\pi}{7}}$$

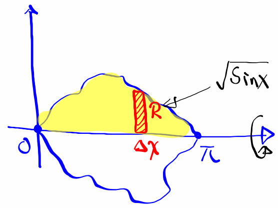
Dec 4-10:37 AM

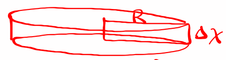
Consider the region below $f(x)=\sqrt{\sin x}$, above x-axis from $x=0$ to $x=\pi$.

$f(0)=0$

$f(\pi)=0$

$0 \le \sin x \le 1$ on $[0,\pi]$





$V=\pi R^2 \Delta x$

Rotate about x-axis

$$V=\int_0^{\pi} \pi \left(\sqrt{\sin x}\right)^2 dx = \pi \int_0^{\pi} \sin x \, dx$$

$\int \sin x = -\cos x + C$

$$= -\pi \cdot \cos x \Big|_0^{\pi} = -\pi\left[\cos\pi - \cos 0\right]$$

$$= -\pi\left[-1-1\right]$$

$$= \boxed{2\pi}$$

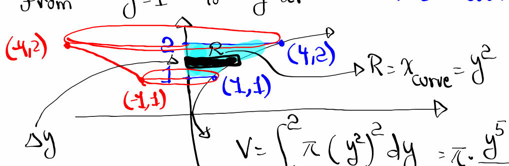
When do we use Disk Method?

1) Region is totally attached to A.O.R.

2) Ref. Rec. is $\perp$ to A.O.R.

Dec 4-10:43 AM

Consider the region enclosed by $x=y^2$, y-axis, from $y=1$ to $y=2$. Rotate about y-axis.



$R = x_{curve} = y^2$

$$V=\int_1^2 \pi \left(y^2\right)^2 dy = \pi \cdot \frac{y^5}{5}\Big|_1^2$$

$$= \frac{\pi}{5}\left[2^5 - 1^5\right]$$

$$= \boxed{\frac{31\pi}{5}}$$

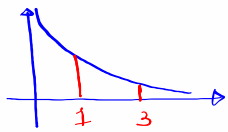
Region is attached to A.O.R.

Ref. Rec. $\perp$ to A.O.R. $\Rightarrow$ Disk Method

Dec 4-10:52 AM

Find f_{ave} for $\underbrace{f(x) = \frac{1}{x^2}}$ on $[1, 3]$

Cont. & integrable on $[1,3]$



$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{3-1}\int_1^3 \frac{1}{x^2}\,dx$$

$$= \frac{1}{2}\int_1^3 x^{-2}\,dx = \frac{1}{2}\cdot \frac{x^{-1}}{-1}\Big|_1^3 = \frac{-1}{2}\cdot\frac{1}{x}\Big|_1^3$$

$$= \frac{-1}{2}\left(\frac{1}{3} - 1\right) = \frac{-1}{2}\cdot\frac{-2}{3} = \boxed{\frac{1}{3}}$$

Dec 4-10:59 AM

Given $f(x) = 2x$, $[0,4]$

Find c in $[0,4]$ Such that $f(c) = f_{ave}$

$f(c) = 2c$

$$f_{ave} = \frac{1}{4-0}\int_0^4 2x\,dx$$

$$= \frac{1}{4}\cdot x^2\Big|_0^4 = \frac{1}{4}(16-0) = 4$$

$f(c) = f_{ave}$

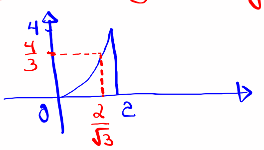
$2c = 4 \rightarrow \boxed{c = 2}$

Dec 4-11:03 AM

Find c in $[0,2]$ such that $f(c) = f_{ave}$ for $f(x) = x^2$. $f(c) = c^2$

$$f_{ave} = \frac{1}{2-0}\int_0^2 x^2\,dx = \frac{1}{2}\cdot\frac{x^3}{3}\Big|_0^2 = \frac{1}{6}\left[2^3 - 0^3\right] = \frac{8}{6} = \frac{4}{3}$$

$$f(c) = f_{ave} \rightarrow c^2 = \frac{4}{3} \rightarrow c = \frac{2}{\sqrt{3}}$$



Dec 4-11:06 AM

$$f(x) = \int_{\boxed{0}}^{\boxed{x}} \frac{t-3}{t^2+7}\,dt$$

Find x-value where $f(x)$ has a a minimum value

$f'(x) = 0$

$f''(x) > 0$

C.U.

Min.

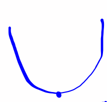
$$f'(x) = \frac{x-3}{x^2+7}\cdot 1 - \frac{0-3}{0^2+7}\cdot 0$$

$$f'(x) = \frac{x-3}{x^2+7} \qquad f'(x) = 0 \rightarrow x-3 = 0 \rightarrow x = 3$$

$$f''(x) = \frac{1\cdot(x^2+7) - (x-3)\cdot 2x}{(x^2+7)^2} = \frac{x^2+7-2x^2+6x}{(x^2+7)^2}$$

$$f''(x) = \frac{7+6x-x^2}{(x^2+7)^2} \qquad f''(3) = \frac{7+6(3)-3^2}{(3^2+7)^2}$$

$$f''(3) > 0 \rightarrow \text{C.U.}$$



at $x = 3$

Minimum value

Dec 4-11:11 AM

$f(x) = \int_0^{\boxed{x}} \frac{1}{1+t^2}\,dt + \int_0^{\boxed{1/x}} \frac{1}{1+t^2}\,dt$ over $(0, \infty)$

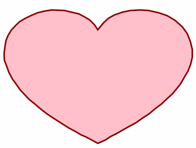
Find $f'(x)$

$$f'(x) = \frac{1}{1+x^2}\cdot 1 - 0 + \frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot \frac{-1}{x^2} - 0$$

$$= \frac{1}{1+x^2} - \frac{1}{\left(1+\frac{1}{x^2}\right)\cdot x^2}$$

$$= \frac{1}{1+x^2} - \frac{1}{x^2+1} = 0 \to f'(x) = 0$$

Since $f'(x) = 0$, then $f(x)$ is a Constant Function



Dec 4-11:19 AM

Evaluate $\int_1^4 \frac{\pi}{\sqrt{x}} \cos\left(\frac{\pi\sqrt{x}}{2}\right) dx$ $\to x^{1/2}$

Hint: Let $u = \frac{\pi\sqrt{x}}{2}$ $x=1$ $x=4$

$du = \frac{\pi}{2}\cdot\frac{1}{2}\cdot x^{-1/2}\,dx$

$du = \frac{\pi}{4}\cdot\frac{1}{\sqrt{x}}\,dx$

$4\,du = \frac{\pi}{\sqrt{x}}\,dx$

$$\int_{\pi/2}^{\pi} \cos u \cdot 4\,du$$

$$= 4\cdot \sin u \Big|_{\pi/2}^{\pi} = 4\left[\sin\pi - \sin\frac{\pi}{2}\right]$$

$$= \boxed{-4}$$

Dec 4-11:24 AM