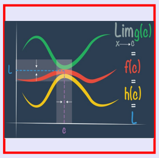
Math 261

Fall 2023

Lecture 50

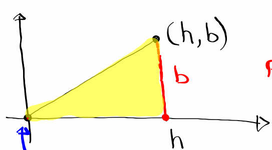


Feb 19-8:47 AM

Given $b>0$, $h>0$, Use Riemann Sum to find the area below $f(x)=\frac{b}{h}x$, above x-axis from $x=0$ to $x=h$.

$f(0)=\frac{b}{h}(0)=0$

$f(h)=\frac{b}{h}(h)=b$



$A=\lim\limits_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x$

$\Delta x=\frac{h-0}{n}=\frac{h}{n}$ ✓

$x_i=0+i\cdot\Delta x=i\cdot\frac{h}{n}=\frac{ih}{n}$ ✓

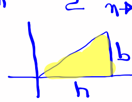
$f(x_i)=\frac{b}{\cancel{h}}\left(\frac{i\cancel{h}}{n}\right)=\frac{bi}{n}$ ✓ $\quad f(x_i)\cdot\Delta x=\frac{bi}{n}\cdot\frac{h}{n}=\frac{bh\,i}{n^2}$ ✓

$\sum_{i=1}^{n} f(x_i)\cdot\Delta x=\sum_{i=1}^{n}\frac{bh\,i}{n^2}=\frac{bh}{n^2}\cdot\sum_{i=1}^{n} i$

$=\frac{bh}{n^2}\cdot\frac{n(n+1)}{2}=\frac{bh}{2}\cdot\frac{n^2+n}{n^2}$

$\lim\limits_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\Delta x=\lim\limits_{n\to\infty}\frac{bh}{2}\cdot\frac{n^2+\cdots}{n^2}=\frac{bh}{2}\cdot\lim\limits_{n\to\infty}\frac{n^2+\cdots}{n^2}$

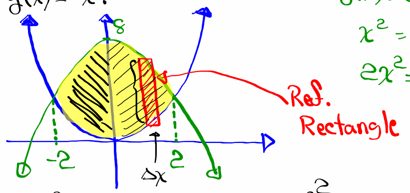
$=\frac{bh}{2}\cdot 1=\boxed{\frac{bh}{2}}$ ✓



$A=\int_0^h \frac{b}{h}x\,dx=\frac{b}{h}\cdot\frac{x^2}{2}\Big|_0^h=\frac{b}{2h}\left[h^2-0^2\right]=\frac{bh^2}{2h}=\boxed{\frac{bh}{2}}$

Nov 29-10:26 AM

Find the area enclosed by $f(x) = 8 - x^2$ and $g(x) = x^2$.



$g(x) = f(x)$

$x^2 = 8 - x^2$

$2x^2 = 8 \qquad x^2 = 4$

$x = \pm 2$

$$A = \int_{-2}^{2} [\text{Top} - \text{Bottom}]\,dx = \int_{-2}^{2} [8 - x^2 - x^2]\,dx$$

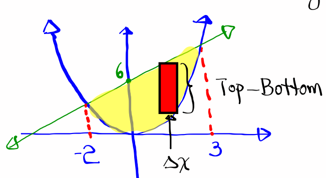
$$= 2\int_{0}^{2} (8 - 2x^2)\,dx = 2\left[8x - \frac{2x^3}{3}\right]\Big|_0^2$$

$$= 2\left[\left(8\cdot 2 - \frac{2\cdot 2^3}{3}\right) - \left(\cancel{8\cdot 0 - \frac{2\cdot 0^3}{3}}\right)\right] \quad 0$$

$$= 2\left[16 - \frac{16}{3}\right] = 2\left[\frac{48-16}{3}\right] = 2\cdot\frac{32}{3} = \boxed{\frac{64}{3}}$$

Nov 29-10:39 AM

Find the area enclosed by $f(x) = x + 6$ and $g(x) = x^2$.



$g(x) = f(x)$

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3 \qquad x = -2$

$$A = \int_{-2}^{3} [x + 6 - x^2]\,dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]\Big|_{-2}^{3} = \left[\left(\frac{3^2}{2} + 6(3) - \frac{3^3}{3}\right) - \left(\frac{(-2)^2}{2} + 6(-2) - \frac{(-2)^3}{3}\right)\right]$$

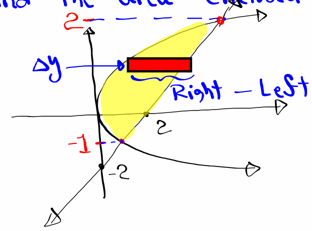
$$= \frac{9}{2} + 18 - 9 - 2 + 12 - \frac{8}{3}$$

$$= \frac{9}{2} + 19 - \frac{8}{3}$$

$$= \boxed{\frac{125}{6}}$$

Nov 29-10:47 AM

Find the area enclosed by $x=y^2$ and $y-x=-2$.



$$y-y^2=-2$$
$$-y^2+y+2=0$$
$$y^2-y-2=0$$
$$(y-2)(y+1)=0$$
$$y=2 \quad y=-1$$

$$y-x=-2$$
$$y+2=x$$

$$A=\int_{-1}^{2}[\text{Line}-\text{Curve}]\,dy$$
$$=\int_{-1}^{2}[y+2-y^2]\,dy$$
$$=\left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]\Bigg|_{-1}^{2}=\boxed{\frac{9}{2}}$$

Please verify this on your own.

Nov 29-10:55 AM

Evaluate $\int_0^2 x\sqrt{4-x^2}\,dx$

Use Subs. method → $u=4-x^2$

$x=0\to u=4$

$du=-2x\,dx$

$x=2\to u=0$

$\frac{du}{-2}=x\,dx$

$$\int_4^0\sqrt{u}\,\frac{du}{-2}$$
$$=-\frac{1}{2}\int_4^0 u^{1/2}\,du=-\frac{1}{2}\cdot\frac{u^{3/2}}{3/2}\Bigg|_4^0$$
$$=-\frac{1}{3}\cdot u\sqrt{u}\Bigg|_4^0=-\frac{1}{3}\left[0\sqrt{0}-4\sqrt{4}\right]$$
$$=-\frac{1}{3}[0-8]=\frac{8}{3}$$

Nov 29-11:04 AM

Evaluate $\int_{\pi/12}^{\pi/9} \sec^2 3x \, dx$

Hint: Use $u=3x$ for Subs. method.

$u=3x$

$du=3dx$

$\frac{du}{3}=dx$

$x=\frac{\pi}{12} \to u=3\cdot\frac{\pi}{12}=\frac{\pi}{4}$

$x=\frac{\pi}{9} \to u=3\cdot\frac{\pi}{9}=\frac{\pi}{3}$

$$=\int_{\frac{\pi}{4}}^{\pi/3} \sec^2 u \, \frac{du}{3}$$

$$=\frac{1}{3}\cdot \tan u \Big|_{\pi/4}^{\pi/3}$$

$$=\frac{1}{3}\left[\tan\frac{\pi}{3}-\tan\frac{\pi}{4}\right]$$

$$=\frac{1}{3}\left[\sqrt{3}-1\right]=\boxed{\frac{\sqrt{3}-1}{3}}$$

Nov 29-11:10 AM

Evaluate $\int_0^{\pi/2} \sin^2 3x \cdot \cos 3x \, dx$

$x=0 \to u=0$

$x=\frac{\pi}{2} \to u=\frac{3\pi}{2}$

$u=3x$

$du=3dx$

$\frac{du}{3}=dx$

$$=\int_0^{3\pi/2} \sin^2 u \cdot \cos u \cdot \frac{du}{3}$$

$$=\frac{1}{3}\int_0^{3\pi/2} \sin^2 u \cdot \cos u \, du$$

Let $w=\sin u$

$dw=\cos u \, du$

$$=\frac{1}{3}\int_0^{3\pi/2} \left[\sin u\right]^2 \cdot \cos u \, du$$

$u=0 \to w=0$

$u=\frac{3\pi}{2} \to w=-1$

$$=\frac{1}{3}\int_0^{-1} w^2 \, dw$$

$$=\frac{1}{3}\cdot\frac{w^3}{3}\Big|_0^{-1}=\frac{1}{9}\left[(-1)^3-0^3\right]$$

$$=\boxed{\frac{-1}{9}}$$

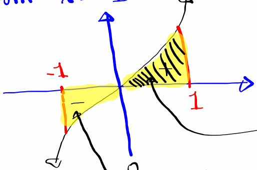
Nov 29-11:16 AM

Evaluate $\int_{-1}^{1} x^3\,dx = \frac{x^4}{4}\Big|_{-1}^{1} = \frac{1}{4}\left[1^4 - (-1)^4\right]$

$= \frac{1}{4}\left[1 - 1\right] = \boxed{0}$

Find the area enclosed by $f(x) = x^3$ & x-axis from $x = -1$ to $x = 1$.

$\cancel{A = \int_{-1}^{1} x^3\,dx}$



$\int_{0}^{1} \left[x^3 - 0\right] dx$

$\int_{-1}^{0} \left[0 - x^3\right] dx$

$A = 2\int_{0}^{1} x^3\,dx = 2 \cdot \frac{x^4}{4}\Big|_{0}^{1} = \boxed{\frac{1}{2}}$