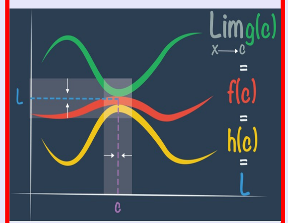
Math 261

Fall 2023

Lecture 5



Class QZ 3

Simplify $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}$, evaluate your final answer with $h=0$.

LCD: $x^2(x+h)^2$

Box Your Final Ans

$$= \frac{x^2\cdot 1-(x+h)^2\cdot 1}{x^2(x+h)^2\cdot h}$$

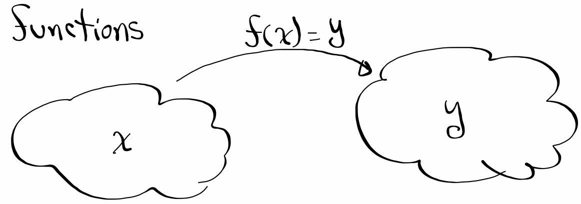
$$= \frac{x^2-\boxed{(x+h)^2}}{x^2(x+h)^2\cdot h} = \frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{x^2(x+h)^2\cdot h} = \frac{-2xh-h^2}{x^2(x+h)^2\cdot h}$$

$$= \frac{\cancel{h}(-2x-h)}{x^2(x+h)^2\cdot\cancel{h}} = \boxed{\frac{-2x-h}{x^2(x+h)^2}}$$

for $h=0$

$$\frac{-2x-0}{x^2(x+0)^2} = \frac{-2x}{x^2\cdot x^2} = \boxed{\frac{-2}{x^3}}$$

Intro. To Functions



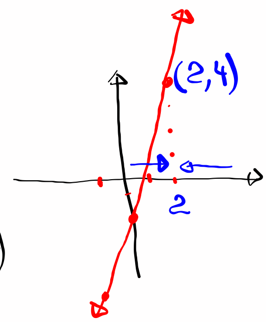
Every x value can have only one y value

$f(x) = 3x - 2$ Linear Function

$f(0) = 3(0) - 2 = -2 \Rightarrow (0, -2)$

$f(2) = 3(2) - 2 = 4 \Rightarrow (2, 4)$

$f(-1) = 3(-1) - 2 = -5 \Rightarrow (-1, -5)$



as $x \to 2$ from right side $\Rightarrow f(x) = y \to 4$

as $x \to 2$ " left side $\Rightarrow f(x) = y \to 4$

$f(x) = \sqrt{x}$

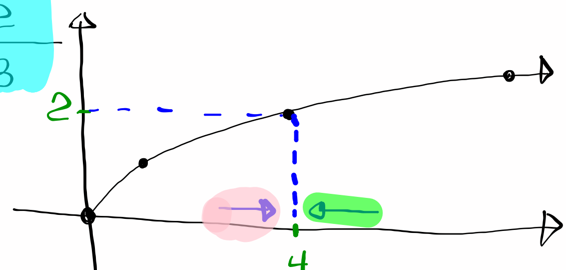
$x \geq 0$

$y \geq 0$

x	y
0	0
1	1
4	2
9	3

Domain $[0, \infty)$

Range $[0, \infty)$



as $x \to 4$ from right, $y \to 2$

as $x \to 4$ " left, $y \to 2$

$\Rightarrow$

as $x \to 4^{+}$, $y \to 2$

as $x \to 4^{-}$, $y \to 2$

$$f(x) = \frac{1}{x-1}$$

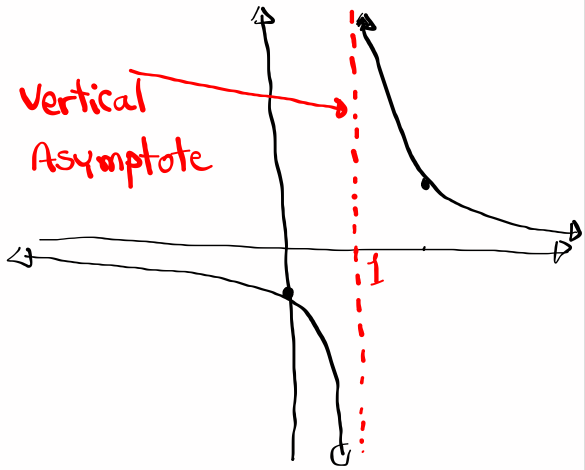
$x - 1 \neq 0 \to x \neq 1$

Domain: $(-\infty, 1) \cup (1, \infty)$

$f(0) = \frac{1}{0-1} = -1$

$f(2) = \frac{1}{2-1} = 1$

Range: $(-\infty, 0) \cup (0, \infty)$

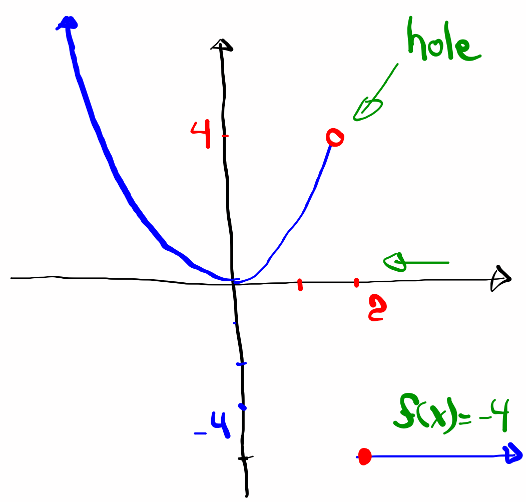


as $x \to 1^+$, $y \to \infty$

as $x \to 1^-$, $y \to -\infty$

Piece-wise function

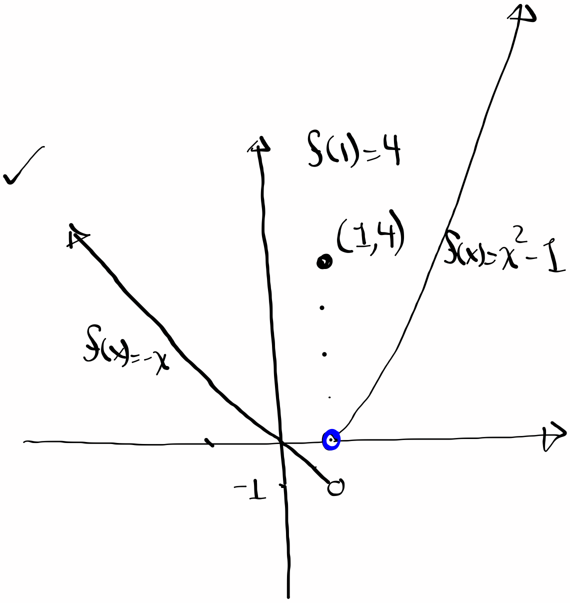
$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ -4 & \text{if } x \geq 2 \end{cases}$$



as $x \to 2^+$, $f(x) = -4$

as $x \to 2^-$, $f(x) \to 4$

$$f(x) = \begin{cases} -x & \text{if } x < 1 \\ 4 & x = 1 \checkmark \\ x^2 - 1 & \text{if } x > 1 \end{cases}$$



as $x \to 1^+$, $f(x) \to 0$

as $x \to 1^-$, $f(x) \to -1$

$f(1) = 4$

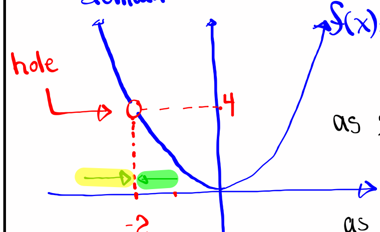
Consider $f(x) = \dfrac{x^3 + 2x^2}{x+2}$

Simplify Yes

modify domain NO

$x+2 \neq 0 \to x \neq -2$

Domain $(-\infty, -2) \cup (-2, \infty)$



$f(x) = \dfrac{x^3 + 2x^2}{x+2} = \dfrac{x^2\cancel{(x+2)}}{\cancel{x+2}} = x^2$

as $x \to -2^+$, $\boxed{f(x) \to 4}$

as $x \to -2^-$, $\boxed{f(x) \to 4}$

$f(-1.99) = \dfrac{(-1.99)^3 + 2(-1.99)^2}{-1.99 + 2} = 3.9601 \to 4$

$f(-1.999) = \dfrac{(-1.999)^3 + 2(-1.999)^2}{-1.999 + 2} = 3.996001 \to 4$

$f(-2.0001) = \dfrac{(-2.0001)^3 + 2(-2.0001)}{-2.0001 + 2} = 4.0004 \to 4$

$f(x) = x^2$

Simplify $\dfrac{f(x+h) - f(x)}{h}$, then evaluate for $h = 0$

Difference quotient

$$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= 2x + h$$

$$\boxed{2x}$$

$f(x) = \sqrt{x}$

Simplify the difference quotient then evaluate for $h = 0$.

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

For $h=0$

$$\frac{\sqrt{x} - \sqrt{x}}{0} = \frac{0}{0}$$

$$= \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

Now evaluate for $h=0$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \boxed{\frac{1}{2\sqrt{x}}}$$

Class QZ 4

Consider $f(x) = \frac{1}{x}$

Simplify the difference quotient, the evaluate for $h=0$. $\boxed{\text{Box Your final Answer.}}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{x-(x+h)}{h\,x(x+h)} = \frac{x-x-h}{hx(x+h)}$$

LCD $= x(x+h)$

$$= \frac{-\cancel{h}}{\cancel{h}x(x+h)} = \boxed{\frac{-1}{x(x+h)}}$$

for $h=0$ $\quad \frac{-1}{x(x+0)} = \frac{-1}{x\cdot x} = \boxed{\frac{-1}{x^2}}$