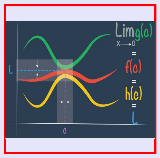
Math 261

Fall 2023

Lecture 48

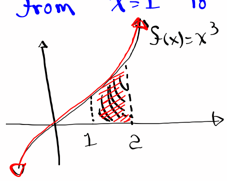


Feb 19-8:47 AM

Class QZ 13

Find the area below $f(x)=x^3$, above x-axis from $x=1$ to $x=2$. Exact Ans. only.

Give some drawing.



$$A=\int_1^2 x^3\,dx = \frac{x^4}{4}\Big|_1^2$$

$$=\frac{1}{4}\left[2^4-1^4\right]=\frac{1}{4}[16-1]=\boxed{\frac{15}{4}}\checkmark$$

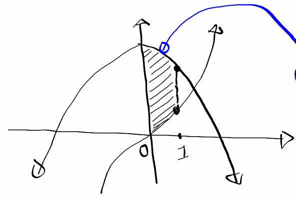
$$=\boxed{3.75}\checkmark$$

Nov 22-11:14 AM

Find the area below $f(x) = 4 - x^2$, above $g(x) = x^3$ from $x = 0$ to $x = 1$

$f(1) = 3$

$g(1) = 1$



$$A = \int_0^1 [\text{Top} - \text{Bottom}]\,dx$$

$$= \int_0^1 [4 - x^2 - x^3]\,dx$$

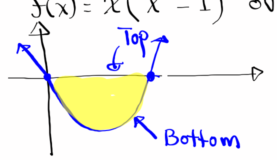
$$= \left(4x - \frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = \left(4(1) - \frac{1^3}{3} - \frac{1^4}{4}\right) - (\ 0)$$

$$= 4 - \frac{1}{3} - \frac{1}{4} = \frac{48 - 4 - 3}{12}$$

$$= \boxed{\frac{41}{12}}$$

Nov 27-10:28 AM

Consider the graph $f(x) = x(x^2 - 1)^3$ over $[0, 1]$



$f(0) = 0$

$f(1) = 0$

$$f(1/2) = \frac{1}{2}\left(\left(\frac{1}{2}\right)^2 - 1\right)^3 = \frac{1}{2}\left(\frac{1}{4} - 1\right)^3 = \frac{1}{2}\left(\frac{-3}{4}\right)^3 = \frac{-27}{128}$$

Find the enclosed area.

$$A = \int_0^1 [\underset{\text{Top}}{0} - \underset{\text{Bottom}}{x(x^2-1)^3}]\,dx = \int_0^1 -x(x^2-1)^3\,dx$$

$$= -\boxed{\int_{0}^{1} x(x^2-1)^3\,dx}$$

$u = x^2 - 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$x = 0 \quad u = -1$

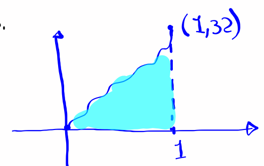
$x = 1 \quad u = 0$

$$= -\int_{-1}^{0} u^3\,\frac{du}{2}$$

$$= -\frac{1}{2} \cdot \frac{u^4}{4}\Big|_{-1}^{0} = \frac{-1}{8}\left[0^4 - (-1)^4\right] = \frac{-1}{8} \cdot (-1) = \boxed{\frac{1}{8}}$$

Nov 27-10:33 AM

Find the area below $f(x) = x^2(x^3+1)^5$ over $[0,1]$, above x-axis.



$f(0) = 0$

$f(1) = 32$

$f(x) > 0$ on $(0,1)$

$$A = \int_0^1 x^2(x^3+1)^5\,dx$$

$u = x^3+1$ $\quad x=0 \quad u=1$

$du = 3x^2\,dx$ $\quad x=1 \quad u=2$

$\frac{du}{3} = x^2\,dx$

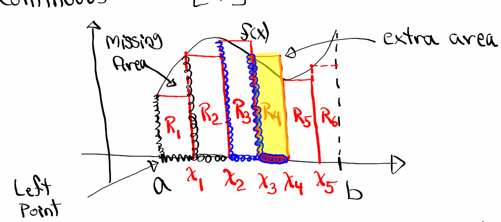
$$= \int_1^2 u^5 \frac{du}{3}$$

$$= \frac{1}{3}\cdot\frac{u^6}{6}\Big|_1^2 = \frac{1}{18}\left[2^6 - 1^6\right] = \frac{64-1}{18}$$

$$= \frac{63}{18} = \boxed{\frac{7}{2}}$$

Nov 27-10:43 AM

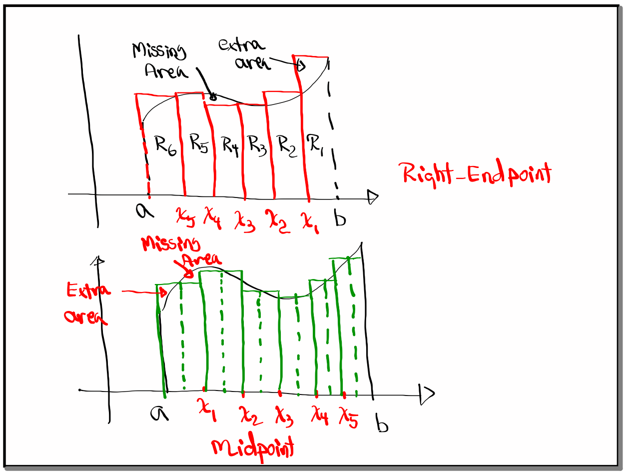
Consider $f(x) \geq 0$ over $[a,b]$ and $f(x)$ is continuous over $[a,b]$



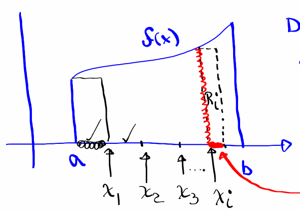
$A(R_1) = f(a)\cdot(x_1 - a)$ $\quad A(R_2) = f(x_1)\cdot(x_2 - x_1)$

$A(R_3) = f(x_2)\cdot(x_3 - x_2)$ $\quad A(R_4) = f(x_3)\cdot(x_4 - x_3)$

Nov 27-10:51 AM



Nov 27-10:59 AM



Divide $[a,b]$ in n subintervals

Length of each subinterval $\frac{b-a}{n}$

$x_1 = a + 1\cdot\frac{b-a}{n}$, $x_2 = a + 2\cdot\frac{b-a}{n}$

$x_3 = a + 3\cdot\frac{b-a}{n}$ ... $x_i = a + i\cdot\frac{b-a}{n}$

$A(R_i) = f(x_i)\cdot\frac{b-a}{n} = f\left(a + i\cdot\frac{b-a}{n}\right)\cdot\frac{b-a}{n}$

Total Area $= R_1 + R_2 + R_3 + \cdots + R_n$

$$= \sum_{i=1}^{n} R_i = \sum_{i=1}^{n} f\Big(\underbrace{a + i\cdot\frac{b-a}{n}}_{x_i}\Big)\cdot\underbrace{\frac{b-a}{n}}_{\Delta x}$$

Delta

Since $n \to \infty$

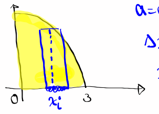
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

If limit exists

$$= \int_a^b f(x)\,dx$$

Nov 27-11:05 AM

Consider $f(x) = 9 - x^2$, find the area below $f(x)$, and above x-axis in QI.



$a = 0$, $b = 3$, $\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{3}{n}$

$x_i = a + i \cdot \Delta x$

$= 0 + i \cdot \frac{3}{n} = \frac{3i}{n}$

$$A(R_i) = f(x_i) \cdot \Delta x = f\left(\frac{3i}{n}\right) \cdot \frac{3}{n}$$

$$= \left[9 - \left(\frac{3i}{n}\right)^2\right] \cdot \frac{3}{n}$$

$$= \left(9 - \frac{9i^2}{n^2}\right) \cdot \frac{3}{n}$$

$$= \frac{27}{n} - \frac{27i^2}{n^3}$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} A(R_i) = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{27}{n} - \frac{27i^2}{n^3}\right)$$

$$= \lim_{n \to \infty} \left[\frac{27}{n} \sum_{i=1}^{n} 1 - \frac{27}{n^3} \sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n \to \infty} \left[\frac{27}{\cancel{n}} \cdot \cancel{n} - \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n \to \infty} \left[27 - \frac{54\cancel{n^3} + \cdots}{6\cancel{n^3}}\right]$$

$$= 27 - \frac{54}{6} = 27 - 9 = \boxed{18}$$

$$A = \int_0^3 [9 - x^2]\,dx = \left(9x - \frac{x^3}{3}\right)\Big|_0^3$$

$$= \left(9 \cdot 3 - \frac{3^3}{3}\right) - (0)$$

$$= 27 - \frac{27}{3} = 27 - 9 = \boxed{18}$$

Nov 27-11:16 AM